

Vibration	Application of Tracking Solver to Eigenvalue Equation	Rotating Machinery
Others		

Object Machine	Complex eigenvalue calculation of shaft and rotor-bearing system of rotating machinery	
Observed Phenomena	<p>As for a three-disk rotor system shown in Fig.1, complex eigenvalue calculations were conducted by using the oil film dynamic characteristics of cylindrical bearings, and the eigenvalue for each rotating speed was plotted on a Gaussian plane as root locus, with one example shown in Fig.2. When drawing root locus of complex eigenvalues as in this example, the following problems arise:</p> <ol style="list-style-type: none"> (1) It is difficult to find where plotting points are smoothly connected as to follow the same solution when the rotational speed varies. (2) In many cases, it is not clear if the root of each mode intersects or not. 	
Cause Estimation	<p>The conventional method of calculating eigenvalues at each rotational speed under a given condition is executed by step by step. Judgement was made that the above method involves ambiguity that cannot be eliminated, thus no further pursuit was made. As an alternative, a new method was conceived wherein, if a correct solution is given as an initial value, afterward, correct solutions may be continuously obtained as a time history response waveform depending upon the increase of the rotational speed. This method ensures that solutions will not deviate from the correct locus, even for gradual changes in the conditions for varying rotational speed.</p>	
Analysis and Data Processing	<p>Attention was paid to the principle of operational of power amplifiers. In Fig.3, an input command v_1 is applied to the operational amplifier (OP amp), then is sent to FET (valve opening) to cause a coil current I (flowrate) to flow. In this circuit, a resistor R_f is inserted, whose voltage $R_f I$ is fed back to the OP amplifier. Since the OP amplifier has a high gain of 10^4, it always acts so that $v_1 - R_f I = 0 \rightarrow I = v_1 / R_f$, thus a current proportional to the input command flows through the coil. That is, OP amplifier functions to cancel errors in the proportional relationship between the input command and the output current, irrespective of the load conditions.</p> <p>The above is applied to the root-finding/following problem of an eigenvalue equation $f(\lambda) = 0$. An error $f(\lambda)$ is applied to OP amplifier and a feedback circuit is configured as in Fig.4. Fig.5 (1) shows the solution of a simple problem solved by this method. Starting roots are continuously obtained, but on the way, a locus changes to another root. Thus, as in Fig.6, the OP amplifier circuit was changed to include a switch Sw. Consequently, it is now possible to grasp/following the same root from the beginning to the end continuously (Fig.5 (2)).</p>	
Countermeasures and Results	<p>Recalculation was made of the complex eigenvalue of the three-disk rotor system supported by the above-mentioned plain bearings. In Figs.7 to 8, loci of the investigated four roots are indicated, starting from the lower one.</p> <ol style="list-style-type: none"> (1) In case of $C/R = 0.001$ (Fig.7): a half-speed mode λ_1 becomes unstable near the twice of natural frequency of bend ω_n, to cause oil whip to occur. (2) In case of $C/R = 0.003$ (Fig.8): a shaft bending mode λ_3 becomes unstable near the twice of natural frequency of bend ω_n, to cause oil whip to occur. <p>Both of the above phenomena generate oil whip, and have a similar mode with the shaft bending mode as principal one, but it was found that they have a different growth. Besides eigenvalue problems, this method may be applied to root-finding problems of various equations.</p>	
Lesson	Electronic circuit engineers say that high-gain feedback is a daily technique. It was realized that, by looking at other fields, many fresh ideas for mechanical engineers might be found.	
References	<ol style="list-style-type: none"> (1) Kikuchi, et al., "Stability analysis of rotating shaft system having multi-bearings and multi-plates", Transaction of the Japan Society of Mechanical Engineers, 43-368 (1977-4), pp.1338-1347 (2) Fujiwara, Matsushita, et al., "Continuous grasping method using a sliding mode control for rotor shaft system eigenvalue problems", Transaction of the Japan Society of Mechanical Engineers C, 71-701 (2005-1), pp.43-50 	
Keywords	Complex eigenvalue, operational amplifier, plain bearing, stability	

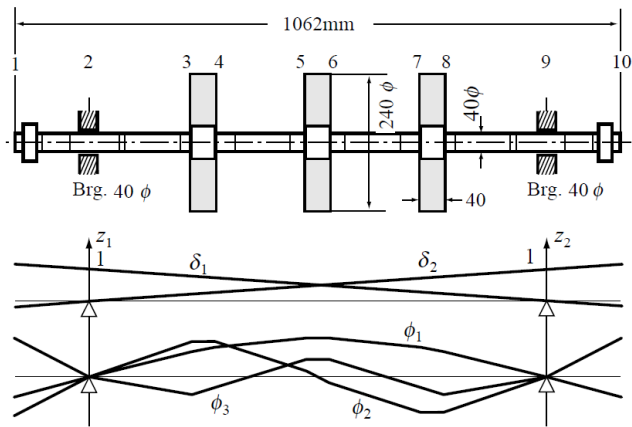


Fig.1 A3-disc rotor-bearing system

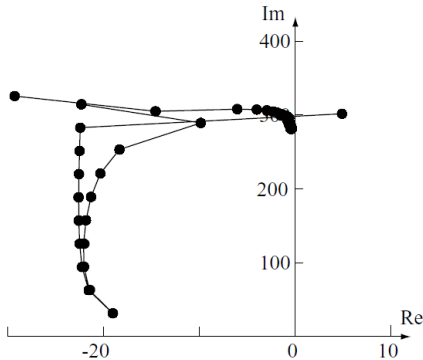


Fig.2 Complex eigenvalues (C/R=0.001)

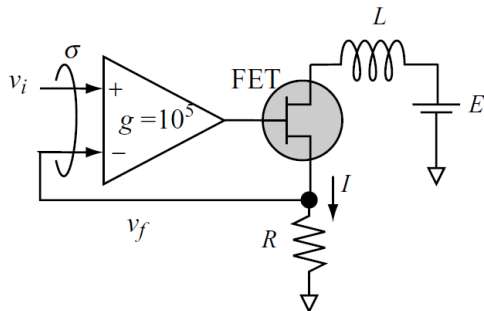


Fig.3 OP amp. feedback circuit

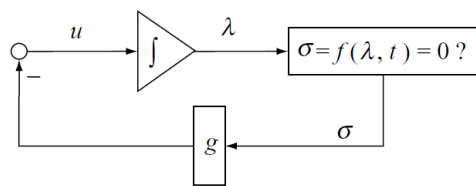
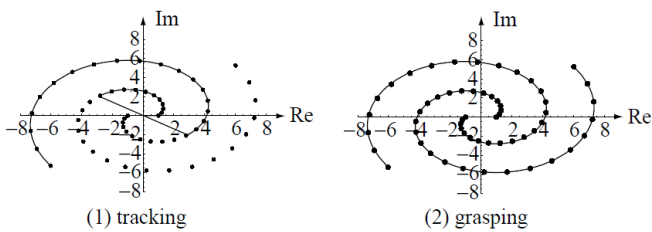


Fig.4 Feedback diagram of OP amp



Note: ● exact, — tenacious solver

Fig.5 Examples: $f(\lambda, t) = 0$

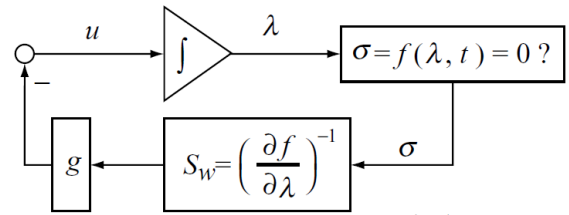


Fig.6 Tracking solver for $f(\lambda, t) = 0$

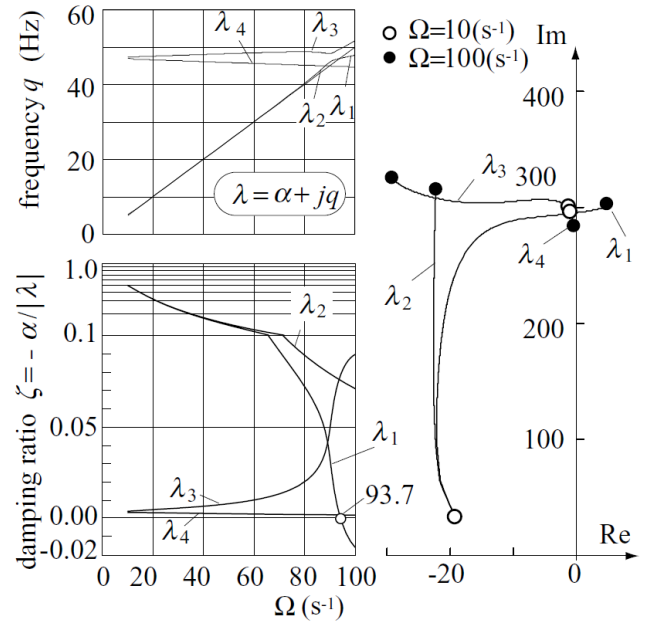


Fig.7 Complex eigenvalues (C/R = 0.001)

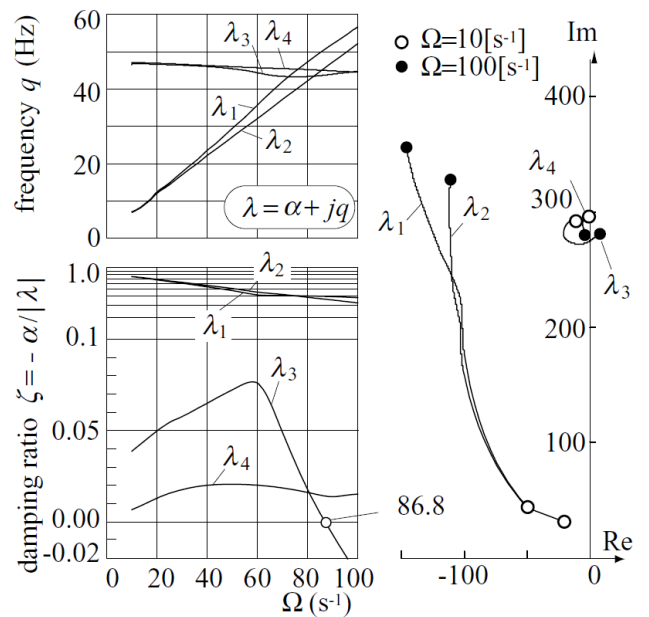


Fig.8 Complex eigenvalues (C/R = 0.003)