MODELING OF FLEXURAL WAVE PROPAGATION IN A PLATE WITH CONTACTING INTERFACES

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1. Background

Recently, the use of nonlinear ultrasonic phenomena such as higher harmonic generation is actively investigated for nondestructive characterization of imperfect interfaces. While the foregoing investigations mainly involve nonlinear features of bulk ultrasonic waves, there is a definite interest in the use of guided wave nonlinearity to characterize the imperfection of bond interfaces. Recently, Shima et al. [1] and Kishiwada et al. [2] experimentally studied the harmonic generation behavior of Lamb wave propagating in a thin plate in solid-solid contact conditions. In particular, it was found in [2] for a plate compressed between solid blocks that the magnitude of odd-order harmonics changed sensitively with the applied contact pressure. Such studies motivate a theoretical analysis for nonlinear guided wave propagation along imperfect interfaces. In this study, the flexural wave propagation in a plate with solid-solid contacting interfaces is modeled using the Mindlin plate theory [3] by incorporating the nonlinear interface model of contacting surfaces [4]. Some numerical examples are shown which demonstrate the wave packet propagation and the higher harmonic generation behavior.

2. Theoretical Modelling

It is known that the dispersion relation of the $A_0$-mode Lamb wave is to a good approximation modeled by the Mindlin plate theory [3]. The plate (thickness $H$) is assumed to be made of an isotropic elastic solid and compressed between rigid planes, and subjected to nonlinear restoring forces at the upper and the lower contacting interfaces. Namely, at each interface, the normal and tangential tractions, $\sigma$ and $\tau$, respectively, are given by

$$\sigma(h,s) = \sigma_0 + K_N(h-h_0) + K_{NN}(h-h_0)^2 + K_{TT}s^2 + K_{NNN}(h-h_0)^3 + K_{NTT}(h-h_0)s^2, \quad (1)$$

$$\tau(h,s) = K_Ts + K_{NT}(h-h_0)s + K_{NNT}(h-h_0)^2s + K_{TTT}s^3, \quad (2)$$

as functions of the normal gap $h$ and the relative tangential displacement $s$, expanded around the equilibrium $h = h_0$ and $s = 0$ up to the third order. In the above expression, $K_N$ and $K_T$ are the linear stiffnesses for normal and tangential directions, and $K_{NN}$, $K_{NT}$, $K_{NNT}$, $K_{NNN}$, $K_{TT}$, $K_{NTT}$, $K_{NT}$, $K_{NNT}$ and $K_{TTT}$ are the expansion coefficients for higher orders. Note that $\sigma$ and $\tau$ have been assumed to be even and odd functions of $s$, respectively. In dynamic motion with plate deflection $w$ and rotation $\phi$, the gap is approximately given by $h = h_0 - w$ and the relative displacement by $s = H\phi/2$ for the
upper interface, and likewise for the lower one (c.f. the insert in Fig. 1). Furthermore, it is assumed here that the upper and lower interfaces are in the same condition of contact. Then, by incorporating the above tractions as the external loading to the plate, the deflection $w$ and the rotation $\phi$ of the plate obey the following differential equations.

$$\rho \frac{\partial^2 w}{\partial t^2} = \kappa G H \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial \phi}{\partial x} \right) - 2 K_N \left\{ 1 + \alpha w^2 + \beta (H\phi)^2 \right\} w,$$

(3)

$$\rho l \frac{\partial^2 \phi}{\partial t^2} = D \frac{\partial^2 \phi}{\partial x^2} - \kappa G H \left( \phi \frac{\partial w}{\partial x} \right) - \frac{H}{2} K_T \left\{ 1 + \gamma w^2 + \delta (H\phi)^2 \right\} H\phi,$$

(4)

where $\rho$ is the density, $G$ the shear modulus, $D$ the bending rigidity, $I = H^3/12$, $\kappa = 5/6$ the shear coefficient. The interfacial properties are characterized by the two linear stiffnesses $K_N$ and $K_T$ and four parameters $\alpha = K_{NNN}/K_N$, $\beta = K_{NTT}/K_N$, $\gamma = K_{NNT}/K_T$, $\delta = K_{TTT}/K_T$ representing the interface nonlinearity. The finite difference method is presently used to solve Eqs. (3) and (4).

3. Numerical Analysis

The numerical model is shown in Fig. 1, where the excitation point is located 530 mm to the left of the contacting region of the plate. The excitation is given as the prescribed displacement in the form of a Gaussian wave packet at 1 MHz, with peak amplitude $w_0$. The transmitted wave is recorded at 30 mm to the right of the contacting region. The material parameters are chosen to

![Fig. 1 Numerical model of a plate with double contacting interfaces.](image)

![Fig. 2 Waveforms for linear interfaces.](image)

![Fig. 3 (a) Phase and (b) group velocities for linear interfaces.](image)
represent an aluminum plate, and different values are assumed for interfacial properties to examine their influence. In the present numerical study, only the parameter $\alpha$ is given non-zero values ($\beta = \gamma = \delta = 0$) since the nonlinearity is considered to arise mainly in the relation between the normal traction and the vertical displacement. The tangential stiffness was given by a relation $K_T = 0.28 \ K_N$ based on an approximate Hertz contact model and a recent experiment [5].

4. Results and Discussion

For a plate with linear interfaces ($\alpha = 0$), the displacement waveforms at the recording point are shown in Fig. 2 for different linear interfacial stiffnesses. It is shown in Fig. 4 that the wave packet becomes slower as the interfacial stiffness increases. The dispersion relation for the linear case can be derived from Eqs. (3) and (4) by assuming the harmonic wave motion, which reveals that the group (phase) velocity decreases (increases) with the interfacial stiffness at 1 MHz as shown in Fig. 3. It is also shown in Fig. 3 that the slope of the velocity-frequency curve becomes steeper at 1 MHz as the stiffness increases. This explains a broadening of the wave packet as the stiffness increases, as shown in Fig. 2. In the nonlinear case, the influence of (a) the third-order
stiffness parameter $\alpha = K_{NNN}/K_N$ and (b) the excitation amplitude $w_0$ is shown in Fig. 4 for $K_N = 20 \text{ kPa/nm}$. It is shown in Fig. 4 that as these parameters increase, the waveform appears slightly distorted from a sinusoidal wave with Gaussian envelope. The amplitude spectra of the waveforms are shown in Fig. 5, which show the generation of odd-order harmonics. As expected from the cubic terms in Eqs. (3) and (4), the amplitude dependence of the third harmonics is in the third power of the excitation amplitude up to a certain level, as shown in Fig. 6.

5. Concluding Remarks

The nonlinear interface model has been incorporated into the Mindlin plate theory in order to model the flexural wave propagation in a plate with contacting interfaces. By solving the derived set of governing differential equations, the wave packet propagation and the harmonic generation behavior have been demonstrated numerically. Other models of contacting interfaces, such as those involving hysteretic nonlinearity or contact damping, can be incorporated likewise into the theory. Such examinations will be insightful when aiming to compare the theoretical analysis to the experimental findings, and constitute a subject for a future study.

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