AXISYMMETRICAL STRESS AND STRENGTH ANALYSIS OF EPOXY-STEEL COMPOSITE CYLINDERS UNDER TORSIONAL LOADS

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INTRODUCTION
Recently, composite parts have widely been used as the performances in the strength, stiffness and toughness increase, and they can lighten the weight of mechanical structures. Fiber-reinforced composites among composite part have higher specific strength and specific stiffness compared with metallic materials. It is easy to be fabricated, but cracks are hard to be extended in the composite parts. Therefore, the interface stress characteristics of the composite parts are important factor. However, they have not yet been researched in detail. Some papers have carried out on the composite joints [1-9]. However, few researchers have been performed on the stress distribution and the strength of composite cylinders which consist of low strength materials and high strength materials [1-5].

In this paper, the interface stress distributions in composite cylinders of epoxy and steel under torsion were analyzed using axisymmetrical theory of elasticity as a two-body contact problem. In the analysis, the stress distributions at the interfaces where the singular stresses occur were discussed. A valid method for estimating the singularity was also proposed.

The analogous tests were conducted to determine the adhesive layer’s shear strength to analogy to the shear critical stress of the composite cylinders. By using two stress singularity parameters obtained from the numerical stress analysis and analogous tests results, a method for estimating the joint strength was proposed. In addition, the experiments were carried out for measuring the rupture torsional loads of the composite cylinders. The numerical results are compared with the experimental results and the FEM results.

THEORITICAL ANALYSIS
The analysis for the model used in this paper is carried out exactly using aximmetrical theory of elasticity as a two-body contact problem. Expanding the stress distribution which applied to the upper end of finite solid cylinder (steel) into Bessel functions, meanwhile combined with boundary conditions of the joint, all unknown parameters can be determined by solving equations. Then, stress distributions at the interface area can be obtained. By changing related parameters’ values, the influencing factors can be examined.

STRESS SINGULARITY PARAMETERS APPROACH
In the analytical results, the stress fields near the upper edge of the interface of the composite cylinders subjected to torsion show singular behaviors. Methods for evaluating the strength at the points using maximum stresses calculated by the numerical analyses are generally not valid. Therefore, a method for estimating the torsional strength is proposed to modify the numerical results near the vicinity of singularity point owing to the singularity stresses.

EXPERIMENTAL METHOD
In this paper, the experimental methods include two parts. One is the analogous tests for shear strength of the composite cylinders under torsion. Another is torsion tests of the composite cylinders under torsion.

**Analogous tests for shear strength**

In this paper, the torsional load $T_w$ is defined as the joint strength when the rupture occurs at the interfaces by the load $T_w$. As shown in Fig.1, the shear stress increases as the torsional loads $T_w$ increase, and the rupture of adhesive layer at the interface is initiated at the end of the interface area ($r=a_2$, $z=h_2$) when the shear stress reaches a critical value. It can be easily assumed that the joint strength increases as the critical value increases. However, from the conventional failure criteria, it is impossible to estimate the joint strength. Thus, the test was carried out to determine the adhesive layer’s rupture strength, and using the test result and the interface stress distribution obtained from the present analyses, the joint strength is predicted in this paper. The joint strength can be estimated by using the shear stress obtained from the numerical analysis.

**Torsion tests of epoxy-steel composite cylinders**

The steel-epoxy composite cylinders were placed on the testing equipment and the maximum torsional loads were measured as the ruptured loads.

The rupture of composite cylinders was observed at the interface where the stress singularity occurred in the numerical calculations.

**FEM METHOD**

In order to verify the numerical results, FEM is used as an approximate method.

**CONCLUSIONS**

In this study, the stress distributions and the strength of composite cylinders subjected to torsional loads are discussed based on the theoretical analysis, the experiments and FEM, and the following results are obtained.

(1) In the composite cylinders of finite solid cylinder and finite hollow cylinder, the interface stress distributions are analyzed as a two-body contact problem using axisymmetrical theory of elasticity. In the numerical calculations, it is found that the shear stress at the upper edge of the interface decreases as $E_1/E_2$ and $b_1/b_2$ increase.

(2) Based on the shear stress obtained from the numerical analysis, it is found that the torsional strength is improved, when 

\[ E_1/E_2 \text{ decreases}, \quad b_1/b_2 \text{ decreases}. \]

(3) The rupture of the adhesive layer is initiated from the upper edge of the interface when torsion is applied to the upper end of the shaft.

(4) A valid method for estimating stress singularity at the vicinity upper edge of the interface of the composite cylinders is proposed to modify the maximum stresses used to evaluate the strength calculated by the numerical analyses.

(5) A method for predict the torsional strength of composite cylinders is demonstrated by the analogous experimental test results.

(6) It is observed that the analytical results are in fairly good agreement with the experimental results and the FEM results under the torsional loads.

\[
T_w = \int_0^{h_2} 2\pi r F(r) dr \tag{1}
\]

\[
\left. \begin{array}{l}
    z_1 = h_1: \tau_{z\theta} = R_0 r + \sum_{s=1}^{\infty} R_s J_1(\alpha_s r) \\
    z_1 = -h_1: \tau_{z\theta} ^{\perp} = 0
\end{array} \right\} \tag{2}
\]
\[
\begin{align*}
 r &= b_1 : r_{r\theta}^l = 0 \quad \left( h_2 \leq |z_1| \leq h_1 \right) \\
 z_2 &= \pm h_2 : r_{z\theta}^l = 0 \\
 r &= b_2 : v_{r\theta}^l = 0 \\
 \left( r_{r\theta}^l \right)_{r = b_1} &= \left( r_{r\theta}^l \right)_{r = a_2} \quad \left( |z_1| \leq h_2 \right) \\
 \left( v_{r\theta}^l \right)_{r = b_1} &= \left( v_{r\theta}^l \right)_{r = a_2} \quad \left( |z_1| \leq h_2 \right) \\
 \nabla^2 \lambda_3 &= 0 \quad \left( \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right) \\
 2 G v_{r\theta}^l &= -\frac{\partial \lambda_3}{\partial r} \\
 \tau_{z\theta} &= -\frac{1}{2} \frac{\partial \lambda_3}{\partial r} \\
 \tau_{r\theta} &= \frac{1}{2} \frac{\partial^2 \lambda_3}{\partial z^2} + \frac{1}{r} \frac{\partial \lambda_3}{\partial r} \\
 G_1 v_{r\theta}^l &= G_1 v_{r\theta}^l + G_1 v_{r\theta}^l + G_1 v_{r\theta}^l \\
 &= G_1 v_{r\theta}^l \left( A_0^l, B_0^l, A_n^l, C_n^l, h_1, h_2, \beta_n^l, \alpha_n^l, G_1, v_{r\theta}^l, r, z_1 \right) + G_2 v_{r\theta}^l \left( A_0^l, C_n^l, h_1, h_2, \beta_n^l, \alpha_n^l, G_1, v_{r\theta}^l, r, z_1 \right) \\
 &= \tilde{A}_0^l r + \tilde{B}_0^l r z + \sum_{n=1}^{\infty} \tilde{A}_n^l \left( \beta_n^l r \right) \sin \left( \beta_n^l z \right) + \sum_{n=1}^{\infty} \tilde{C}_n^l \sinh \left( \alpha_n^l z \right) J_1 \left( \alpha_n^l r \right) \\
 &+ \sum_{n=1}^{\infty} \tilde{A}_n^l \left( \beta_n^l r \right) \cos \left( \beta_n^l z \right) + \sum_{n=1}^{\infty} \tilde{C}_n^l \cosh \left( \alpha_n^l z \right) J_1 \left( \alpha_n^l r \right) + \tilde{A}_0^l r^2 + \tilde{B}_0^l r^3 \\
 G_2 v_{r\theta}^l &= G_2 v_{r\theta}^l + G_2 v_{r\theta}^l + G_2 v_{r\theta}^l \\
 &= G_2 v_{r\theta}^l \left( A_0^l, B_0^l, A_n^l, B_n^l, C_n^l, a_2, h_2, \beta_n^l, \alpha_n^l, G_2, v_{r\theta}^l, r, z_2 \right) + G_2 v_{r\theta}^l \left( A_0^l, B_n^l, C_n^l, a_2, b_2, h_2, \beta_n^l, \alpha_n^l, G_2, v_{r\theta}^l, r, z_2 \right) \\
 &= \tilde{A}_0^l r + \tilde{B}_0^l r z + \tilde{A}_0^l r z^2 + \tilde{B}_0^l r^3 + \tilde{C}_0^l + \sum_{n=1}^{\infty} \tilde{A}_n^l \left( \beta_n^l r \right) \sin \left( \beta_n^l z \right) \\
 &+ \sum_{n=1}^{\infty} \tilde{C}_n^l \sinh \left( \alpha_n^l z \right) C_1 \left( \alpha_n^l r \right) + \sum_{n=1}^{\infty} \tilde{A}_n^l \left( \beta_n^l r \right) + \tilde{B}_0^l K_1 \left( \beta_n^l r \right) \cos \left( \beta_n^l z \right) \\
 \tau (r) &= K / r^2 \\
 Q &= \cos(\lambda \pi) - \frac{1}{1 + l^2} = 0 \\
 \tau_0 &= \tau_a 
\end{align*}
\]
References


Figure 1 A model for analysis of composite cylinders subjected to torsion loads
Figure 2 Effect of Young’s modulus of finite solid cylinder on interface stress distributions

Figure 3 Effect of the diameter of finite solid cylinder on interface stress distributions

Figure 4 Material parameters of a contact structure

Figure 5 Results of the characteristic equation
Figure 7 Analogous tests for obtaining $\tau_{r0}$

Figure 8 Experimental results from analogous tests and the expression’s results from stress singularity

Figure 9 Dimensions of specimens and a schematic experimental setup for torsion tests
Figure 6 Shear stresses close to the singular point

Figure 10 Comparison between the analytical and the experimental results

Figure 11 An example of the elemental division

Figure 12 Comparison between the analytical and the FEM results