Fractal Characteristics of Turbulent Premixed Flame in a Closed Vessel and a Spark-Ignition Engine

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ABSTRACT

The cross-sectional images of turbulent premixed flames were measured in a closed vessel and a spark-ignition engine with homogeneous mixture of fuel and air. The fractal analysis was performed for the flame boundary lines. The fractal characteristics such as fractal dimension, inner cutoff scale and outer cutoff scale were determined, and the relationship among the fractal characteristics, turbulence characteristics and laminar burning characteristics was discussed. The fractal dimension was expressed as functions of the non-dimensional turbulence intensity and the density of unburned gas. The outer cutoff scale was consistent with the equivalent radius. The non-dimensional inner cutoff scale was proportional to \( K_a^{-0.5} \), where \( K_a \) is Karlovitz number.

INTRODUCTION

In a wrinkled laminar flame regime, if the local burning velocity is assumed to be laminar burning velocity, the turbulent burning rate can be expressed as the increase of flame surface based on a fractal geometry concept. After Gouldin(1) proposed the burning velocity model using fractal geometry concept, several researchers have studied the fractal characteristics of turbulent premixed flames and have proposed fractal burning models. In order to formulate the fractal burning model, the fractal characteristics such as fractal dimension, inner cutoff scale and outer cutoff scale must be determined. Gouldin(1) has suggested that the inner cutoff scale is modeled as the function which accounts for an increase in the inner cutoff scale with decreasing \( \frac{u'}{S_L} \) within the range of turbulence scale from \( \eta \) to \( L \), where \( u' \), \( S_L \), \( \eta \) and \( L \) denote turbulence intensity, unstretched laminar burning velocity (entrainment velocity of unburned mixture into the flame front), Kolmogorov scale and integral scale, respectively. In his chemical closure model(2), the fractal description of flame surface is applied to the flame wrinkling smaller than the integral scale of turbulence, and the effect of the flame wrinkling larger than the integral scale on the burning rate per unit volume is accounted for by the probability of finding a flamelet in a small volume. On the other hand, Murayama et al.(3) showed that the inner and outer cutoff scales were equal to laminar flame thickness and the burner size, respectively and that the observed fractal-like character was not directly related with the turbulence characteristics of the approach flow. Recently, Yoshida, et al.(4) showed that the inner cutoff scale was related with the wavelength of laminar flame instability and the outer cutoff scale coincides with the base diameter of conical turbulent flame. These results related above were obtained from measurements on the turbulent premixed flame of the Bunsen burner in atmospheric pressure. The values of the fractal dimension measured in these flames were smaller than the values between 2.32 and 2.40 in non-reacting turbulent flows(5).

There were few studies for the fractal characteristics of turbulent flame propagating in the spark-ignition engines (6,7,8). Mantzaras, et al.(6) reported that the fractal dimension increased with increasing \( \frac{u}{S_L} \) and it approached the value of 2.36 when \( \frac{u}{S_L} \) was extremely larger than unity. Santavicca, et al.(7) have proposed the relationship among the fractal dimension, the turbulence intensity and the laminar burning velocity. Matthews et al.(8) applied the Gouldin's flame speed model(1) to quasi-dimensional engine simulation. The fractal characteristics of turbulent premixed flame were summarized by Gülder (9).

We investigated the turbulent premixed flames propagating in a closed vessel and a spark-ignition engine. The cross sectional flame images in the chambers were measured under the atmospheric condition and the conditions of high temperature and pressure, and the fractal characteristics of the turbulent premixed flame were discussed.

EXPERIMENTAL PROCEDURE

A cylindrical vessel with acrylic windows was used as shown in Fig. 1. The inner diameter and the
width of the chamber were 85 mm and 20 mm, respectively. Under the initial condition, the pressure in the mixture tank was 150 kPa and that in the chamber was 20 kPa. When the solenoid valve was opened, the swirling flow was generated in the chamber. Figure 2 shows the decay of the mean velocity, \( U \), and turbulence intensity, \( u' \), after the valve closing. If the mixture was ignited at a certain time during the decay of flow, the turbulent premixed flames under the several condition of turbulence were produced.

A bottom view engine of four-stroke cycle as shown in Fig. 3 was also used in the experiment. Both bore and stoke were 85 mm, and the top clearance was 24.5 mm. The swirling flow was generated in a cylinder by the shrouded valve. No squish flow was produced because the configuration of the combustion chamber was pancake type. In the previous study (10), the swirl velocity in the cylinder was measured at the top dead center, and the distribution of velocity was found to be similar to the solid body vortex as shown in Fig. 4. Operating conditions of the test engine are shown in Table 1. Propane or methane was used as the fuel. The fuel and air were mixed homogeneously with a static mixer equipped in the upstream of the intake port. The mixture was ignited at the center of the cylinder, after the residual burned gas was removed sufficiently, in order to exclude the effect of residual gas.

For the laser sheet illumination, a pulsed Nd:YAG laser was employed. This laser had a pulse duration of 8 ns and a power of 155 mJ at 532 nm of second harmonic wavelength. The laser sheet had a thickness of 0.2 mm and a width of 50 mm. The mean diameter of the seeding particles, TiO\(_2\), was 0.25 \( \mu \)m. The scattered light was imaged using a CCD camera, and recorded in the frame memory of

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Fig. 1 Experimental apparatus

Fig. 2 Mean velocity and turbulent intensity in a closed vessel

Fig. 3 Test engine with optical assemblies
Table 1 Operating conditions of test engine

<table>
<thead>
<tr>
<th>Fuel</th>
<th>Methane</th>
<th>Propane</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalence ratio</td>
<td>1.0, 0.8</td>
<td>1.0, 0.8, 0.7</td>
</tr>
<tr>
<td>Engine speed</td>
<td>1000</td>
<td>1000, 750</td>
</tr>
<tr>
<td>Compression ratio</td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>Ignition timing</td>
<td>30 deg. BTDC</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 4 Mean velocity and turbulence intensity in a spark ignition engine

- $n = 1000 \text{ rpm}$
- $n = 750 \text{ rpm}$
- $\eta_v = 0.5$

$\overline{(U)}_r, \text{ m/s}$

$t = 6.1 \text{ ms}$
$t = 8.1 \text{ ms}$
$t_{ig} = 0 \text{ ms}$

$t = 7.7 \text{ ms}$
$t = 10.5 \text{ ms}$
$t_{ig} = 40 \text{ ms}$

Fig. 5 Flame images in a closed vessel ($\phi = 1.0$, Methane-air)

- White and black regions in the flame images indicate unburned gas and burned gas, respectively.
- Flame boundaries at the early ignition timing ($t_{ig} = 0 \text{ ms}$) are more complicated than those at the late ignition timing ($t_{ig} = 40 \text{ ms}$). As the flame develops in the chamber, the flame wrinkling becomes the small structure.

CROSS-SECTIONAL FLAME IMAGES

Figure 5 shows the cross-sectional flame images in the closed vessel. The parameters of $t_{ig}$ and $t$ denote ignition timing and timing of laser shot, respectively. The flame images indicate unburned gas and burned gas, respectively. Flame boundaries at the early ignition timing ($t_{ig} = 0 \text{ ms}$) are more complicated than those at the late ignition timing ($t_{ig} = 40 \text{ ms}$). As the flame develops in the chamber, the flame wrinkling becomes the small structure.

The flame boundary lines were detected by digitizing the flame images. The length of the flame boundary line was measured by the circle method(5), and the fractal characteristics were determined. The flame images rather than 20 images was taken for each experimental conditions. The fractal dimension $D_2$ is the averaged value. In these images (Fig. 5 and Fig. 6), the inner cutoff scale was not observed because the distance of each pixel (0.14 mm) was too large to detect the inner cutoff scale.

The fractal dimension has been often expressed as the function of the ratio of the turbulence intensity to the laminar burning velocity, $u'/S_l$ (3,4,6-8). Figure 7 shows the relationship between $D_2$ and $u'/S_l$. Figure 7 indicated that the fractal dimension couldn’t be expressed by one parameter of $u'/S_l$. In order to account for the effect of the density on the fractal dimension, the relationship between $D_2$ and $(\rho/\rho_0)(u'/S_l)$ were plotted in Fig. 8, because the density in the engine was larger than that in the closed vessel. As shown in Fig.8, the fractal dimension is expressed as Eq. (1).
Fig. 6 Flame images in a spark-ignition engine  
(n=1000rpm, Propane-air)

\[
D_2 = \frac{2.0}{1 + \left(\frac{\rho_u}{\rho_0} \right)^2 \left(\frac{u' / S_L}{u'/S_L}\right)^2} + \frac{D_{2m}}{1 + \left(\frac{\rho_u}{\rho_0} \right)^m \left(\frac{u' / S_L}{u'/S_L}\right)^n a} \tag{1}
\]

where \(\rho_u\) is the unburned gas density, and the subscript 0 denotes the reference condition which corresponds the pressure of 101.3 kPa and the temperature of 300 K. Tamura et al. (11) concluded that the nondimensional burning velocity \(S_i / S_L\) was expressed as the function of \((\rho_u / \rho_0)(u' / S_L)\) under high pressure condition. Therefore, the fractal dimension that characterizes the degree of flame roughness should include the effect of the unburned gas density. In the case of \(D_{2m} = 1.30 \sim 1.35\), most of data are expressed by Eq. (1). However the data under the condition of relatively high pressure in the closed vessel are consistent with the values predicted in case of \(D_{2m} = 1.20\). In order to express all of data, fractal dimension were approximated by the following equation.

\[
D_2 = \frac{10}{1 + \left(\frac{\rho_u}{\rho_0}\right)^m \left(\frac{u' / S_L}{u'/S_L}\right)^n a} + \frac{D_{2m}}{1 + \left(\frac{\rho_u}{\rho_0}\right)^m \left(\frac{u' / S_L}{u'/S_L}\right)^n a} \tag{2}
\]

where \(m=4, n=2, a=3\), \(D_{2m}=1.30\). The curve in Fig. 9 indicates the values of Eq(2). The effect of the ratio of gas density, \(\rho_u / \rho_0\), on the fractal dimension \(D_2\) is larger than that of non-dimensional turbulence intensity, \(u'/S_L\).

INNER CUTOFF SCALE AND OUTER CUTOFF SCALE

The outer cutoff scale and the inner cutoff scale represent the largest scale and the smallest scale of wrinkled laminar flame respectively. Figure

Fig. 7 The effects of \(u'/S_L\) on \(D_2\)

Fig. 8 The effects of \((\rho_u / \rho_0)(u' / S_L)\) on \(D_2\)
10 shows the relation between outer cutoff scale and equivalent flame radius. The outer cutoff scale is consistent with the equivalent radius. Several inner cutoff scale formulations based on experimental data or numerical simulation results have been reviewed by Gülder et al.(9). This scale is expressed as the scale of turbulence, or the function of \( u'/S_L \), and the scale of turbulence. Gülder et al.(12) proposed the following equation, assuming that fractal inner cutoff, \( \varepsilon_i \), is proportional to the Kolmogorov scale of turbulence,

\[
\varepsilon_i/\delta = K_a^{-0.5}
\]

(3)

where \( K_a \) is the Karlovitz number defined as \( K_a = (u'/\delta)(L/\delta) \). \( \delta \) and \( \lambda \) denote the preheat zone thickness of laminar flame and Taylor microscale of turbulence. Roberts et al.(13) proposed that the minimum vortex diameter \( d_c \) which wrinkles the flame front is expressed as

\[
d_c/\delta = 2.0(u'/S_L)^{0.75}(L/\delta)^{0.25},
\]

where \( \delta = 7.4\delta \) and \( L \) is the integral scale of turbulence. If \( \varepsilon_i = d_c/2 \), this equation would be converted to the following equation,

\[
\varepsilon_i/\delta = 2.5K_a^{-0.5}
\]

Figure 11 shows the relationship between the non-dimensional inner cutoff scale \( \varepsilon_i/\delta \) and Karlovitz number \( K_a \), based on the experimental results of the closed vessel, with reference to Eq. (3). In this experiment, methane-air mixture was used, and equivalence ratio was varied within the range from 0.7 to 1.2. The following equation was obtained in the range 0.5 < \( u'/S_L \) < 1.5,

\[
\varepsilon_i/\delta = 5K_a^{-0.5}
\]

(4)

From this equation, for \( n=1000 \text{rpm} \), the inner cutoff scale is predicted as 0.7 mm. Figure 12 shows the flame images obtained from the small view field (2 cm x 2 cm), the flame boundary line and the fractal analysis results. As the distribution of the scattering particles is not completely homogeneous, the flame boundary line presents a very small roughness like image noise. However, the wrinkled scale of flames less than 1 mm can be observed. Although the fluctuation of inner cutoff scale is comparatively large in each image, the inner cutoff scale obtained here is within the range of 0.25-0.75 mm, which is probably equal to the value obtained from Eq. (4).

CONCLUSIONS

The fractal characteristics of turbulent premixed flame propagating in a closed vessel and an engine cylinder were discussed. The main results are as follows:
The fractal dimension increases with increasing turbulence intensity and gas density.

The fractal dimension is expressed as a function of $\rho u/\rho_0$ and $u'/S_L$.

The nondimensional inner cutoff scale is proportional to $K_a^{-0.5}$.

The outer cutoff scale is consistent with the equivalent flame radius.

REFERENCES