Penetration Model of a Diesel Spray along a Wall

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ABSTRACT

Behavior of diesel spray impinging on a wall was experimentally and theoretically investigated for further understanding of a mixing process between diesel spray and surroundings. The behavior of diesel spray impinging on a wall was so complex that a simple method to estimate the spray growth characteristics has been needed. To make a simple empirical formula of impingement spray tip penetration, a new model of spray movement was proposed. To introduce the new model, spray growth characteristics were observed by high speed photographs. A sectional view of spray was visualized by a laser sheet light and analyzed to observe the high-density region of spray. The empirical formula was modeled using the spray path length. The empirical formula showed well agreement with spray tip penetration along the wall.

INTRODUCTION

In a small type diesel engine, the injected fuel spray impinges on the piston cavity due to the short distance between an injection nozzle and cavity wall. Therefore, it is important to understand the impingement spray behavior for improving engine performance.

Impingement of diesel spray on a wall was experimentally studied\(^{(1,2)}\) and was numerically analyzed\(^{(3,4)}\) by the KIVA code using submodels for droplet impingement. Influence of diesel spray impingement on vaporization and combustion was reported by Suzuki et al.\(^{(5)}\) and Takasaki et al.\(^{(6)}\) They pointed out that the impingement of a diesel spray caused an increase of the volume of vapor and flame. However, more information about the movement and structure of a diesel spray impinging on a wall is needed to understand these wall effects. The authors had previously presented that a diesel spray penetrating along a wall was deflected by the Coanda effect\(^{(7)}\). Further, for the case of a spray moving parallel or shallowly impinging on a wall (\(\phi = 4\ deg\)), its penetration was a little longer than that of a free spray\(^{(8)}\).

In this study, the development and the internal structure of impingement spray were experimentally observed. The spray path length proposed in the previous paper\(^{(9,10)}\) was used as an unique parameter for the evaluation of the impingement spray development. Further, this paper presents a new model for impingement spray growth characteristics. This model was based on the results of experimental research for free spray and the results of the theoretical research for wall jet. An empirical formula was expressed as a function of impingement distance and impingement angle. It showed well agreement with the measurements.

NOMENCLATURES

\(A_{pre}\) : proportional coefficient for pre-breakup spray
\(A_{post}\) : coefficient for post-breakup spray
\(c\) : experimental coefficient
\(D\) : diameter of injection nozzle
\(L_b\) : breakup length
\(L_{path}\) : spray path length
\(\tilde{L}_{path}\) : path length from impingement
\(L_c\) : impingement distance
\(\tilde{L}_{uni}\) : length of uni-velocity region
\(L_{tip}\) : spray tip penetration
\(t_b\) : breakup period
\(t_{inj}\) : time from injection start
\(t_{imp}\) : time from impingement
\(\tilde{t}_{uni}\) : duration of uni-velocity region
\(t_a\) : impingement time
\(u_0\) : injection velocity of fuel
\(V_{path}\) : growth velocity of spray path

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INTERNAL STRUCTURE OF SPRAY

Lateral images of impingement spray.

The tomographic images of spray taken by a laser sheet were shown in Fig. 2. The inclined wall location was 30 mm but impinging angle was changed from 10 to 90 degrees. All the photographs were taken at the time of 1.5 milliseconds from the start of injection. Free spray(a) showed almost symmetrical pattern. Shallowly impingement spray had a thin layer near the impingement point. Deep or vertical impingement spray stretched concentricity from impingement point.

High density ridges in impinging spray.

Tomographic image of the spray was analyzed to extract the high density ridges in the spray image intensity. Outline of the method was shown in Fig.3. The detailed process of this analysis was described in the previous paper. The high density ridges of spray were clearly extracted as the dotted points in the figure. We called it skeleton of a spray.

Contour shapes and skeletons of shallowly impinging sprays at various timings were shown in Fig.4. Wall parameters were \( L_w = 10 \text{ mm} \) and \( \phi = 4 \text{ deg} \). After a short period of time \( t_{on} = 0.2 \text{ ms} \), coherent structure began to appear in the tip region. It was similar to the free spray. However, they did not grow into the branch components. Impinged liquid column looked like slipping on the wall to the downhill direction. A linear structure of skeleton was located near the wall and the spray grew with forming smooth shape in this region. We defined it as a slip region of impingement spray. Solid lines vertical to wall were added to indicate the tip of this slip region. Length of this region was measured and used in later section. The branch components of skeleton were found at \( t_{on} = 1.5 \text{ ms} \) in the spray tip region.

SPRAY PATH LENGTH

Definition of the spray path length of an impingement spray was shown in Fig.5. The spray path length\(^{10}\) was an imaginal spray tip length which
was derived by the incremental movement of \( L_x \) and \( L_y \) and it was expressed by eq.(1).

\[
L_{path} = \int \sqrt{\left( \frac{dL_x}{dt} \right)^2 + \left( \frac{dL_y}{dt} \right)^2} \, dt
\]  

(1)

This spray path length could present the scalar length along the spray movement before and after impingement.

The spray tip penetration \( L_x \) and spray path length for an impingement spray \( L_w = 30 \text{ mm and } \phi = 30 \text{ deg} \) are shown in Fig.6. \( L_{path} \) showed qualitatively well agreement with \( L_x \). Another coordinate which was oriented on the impingement time and position was added in this figure. Penetrations on this coordinate indicated the penetrations from the imaginary origin set on the impingement point shown in Fig.5. The slip zone length \( L_{syn} \) measured in Fig.4 are also shown in this figure. The length of slip zone increased with time but after a short period of time, it reached at almost constant value.

Logarithmic expression of spray path length measured from the impingement point was shown in Fig.7. This figure shows the linear relationship with two different slopes. At the first stage of spray development along the wall, the slope is about 1. It meant that the spray path length grew in proportion to the time from impingement. In the second stage of spray development, the increment slope of the path length changed from 1 to 1/2.

**MODELING OF A GROWTH BEHAVIOR OF IMPINGEMENT SPRAY**

**Empirical formula of a free spray**

The previous paper\(^{[9]}\) showed that the spray path length of a free spray was a little longer than the traditional spray tip penetration. However, the difference between these two penetrations was small. So the empirical formula of free spray\(^{[11]}\) should be adaptive for \( L_{path} \). The spray tip penetration grew in proportion to the time from the injection start. Then, the path length of pre-breakup spray could be expressed by the linear function of time.

\[
L_{path} = \lambda_{pre} t_{eff} \quad (L_{path} < L_b)
\]  

(2)

Here, parameter \( L_b \) indicated the breakup length of fuel column\(^{[11]}\). The tip penetration of post-breakup spray was proportional to the square root of the time.
\[ L_{\text{path}} = A_{\text{pre}} \left( t_{\text{inj}} \right)^{\frac{1}{2}} \quad (L_{\text{path}} \leq L_b) \]  \tag{3}

Coefficient \( A_{\text{pre}} \) and \( A_{\text{free}} \) were concretely proposed by Arai et al.\cite{12} for the traditional spray tip penetration. They were shown as follows.

\[ A_{\text{pre}} = u_0 = c \frac{2\Delta P}{\rho_l} \quad (L_{\text{path}} < L_b) \]  \tag{4}

\[ A_{\text{free}} = \frac{1}{2} \left( \frac{2\Delta P}{\rho_a} \right)^{\frac{1}{4}} \quad (L_{\text{path}} \geq L_b) \]  \tag{5}

The difference between spray tip penetration and spray path length was small. So these coefficients were used to express the spray path length in the present model. Then, \( L_{\text{path}} \) was expressed as follows.

\[ L_{\text{path}} = u_0 t_{\text{inj}} = c \frac{2\Delta P}{\rho_l} t_{\text{inj}} \quad (L_{\text{path}} < L_b) \]  \tag{6}

\[ L_{\text{path}} = \left( \frac{2\Delta P}{\rho_a} \right)^{\frac{1}{4}} \left( t_{\text{inj}} \right)^{\frac{1}{2}} \quad (L_{\text{path}} \geq L_b) \]  \tag{7}

The duration of breakup process \( t_b \) and the breakup length \( L_b \) were respectively shown as follows.

\[ t_b = \frac{\alpha \rho_l D}{2c^2 \rho_a \Delta P} \]  \tag{8}

\[ L_b = A_{\text{pre}} t_b = \frac{\rho_l}{\rho_a} \]  \tag{9}

Here, \( \alpha \) and \( c \) were experimental coefficients.

**Spray modeling in an uni-velocity zone just after impingement**

The injected fuel made a liquid column and disintegration of it was suppressed before \( t_b \). The momentum exchange caused by the mixture formation process was little in this period. As the results, the spray tip penetration grew in proportion to the elapsed time. We applied these concepts to the impingement spray. In a short period of time from impingement, a spray path length increased in proportion to the elapsed time from impingement as mentioned above. Then, the growth behavior of spray just after impingement was modeled as the uni-velocity penetration which did not coupled with any momentum exchange.

First of all, the velocity of a free spray was derived from differentiation of spray path length expressed by eqs. (6) and (7).

\[ V_{\text{path}} = c \frac{2\Delta P}{\rho_l} \quad (L_{\text{path}} < L_b) \]  \tag{10}

\[ V_{\text{path}} = \frac{1}{2} \left( \frac{2\Delta P}{\rho_a} \right)^{\frac{1}{4}} \left( t_{\text{inj}} \right)^{-\frac{1}{2}} \quad (L_{\text{path}} \geq L_b) \]  \tag{11}

Where, an impingement distance \( L_w \) separated the velocity into two forms. Impingement velocity at the spray during the breakup process was expressed by eq.(12). And equation(13) was the impingement velocity after breakup process.

\[ V_{\text{path}}|_{L_w} = c \frac{2\Delta P}{\rho_l} = A_{\text{pre}} \quad (L_{\text{path}} < L_b) \]  \tag{12}

\[ V_{\text{path}}|_{L_w} = \frac{1}{2} \left( \frac{2\Delta P}{\rho_a} \right)^{\frac{1}{4}} \left( t_{\text{w}} \right)^{-\frac{1}{2}} \quad (L_{\text{path}} \geq L_b) \]  \tag{13}

Here, the relationship between \( t_w \) and \( L_w \) was expressed as follows.

\[ t_w = \frac{L_w}{A_{\text{pre}}} = \frac{L_w}{c} \frac{\rho_l}{2\Delta P} \quad (L_{\text{path}} < L_b) \]  \tag{14}

\[ t_w = \frac{L_w^2}{A_{\text{free}}} = \frac{L_w^2}{\rho_a \Delta P} \quad (L_{\text{path}} \geq L_b) \]  \tag{15}

We supposed that the impinging spray started from the imaginary origin. Then, the velocity of the uni-velocity slip zone was assumed to be consisted with the impingement velocity expressed by eqs.(12) and (13).

\[ V_{\text{uni}} = V_{\text{path}}|_{L_w} = A_{\text{pre}} \quad (L_{\text{path}} < L_b) \]  \tag{16}

\[ V_{\text{uni}} = V_{\text{path}}|_{L_w} = \frac{1}{2} A_{\text{free}} \left( t_w \right)^{-\frac{1}{2}} \quad (L_{\text{path}} \geq L_b) \]  \tag{17}

Therefore, the spray path length during an uni-velocity region was expressed as follows.

\[ L_{\text{path}} = L_w + V_{\text{uni}}(t_{\text{inj}} - t_w) \]  \tag{18}

Equations(16) and (17) were substituted for eq.(18). Then,

\[ L_{\text{path}} = L_w + A_{\text{pre}}(t_{\text{inj}} - t_w) \quad (L_{\text{path}} < L_b) \]  \tag{19}

\[ L_{\text{path}} = L_w + \frac{1}{2} A_{\text{free}} \left( t_w \right)^{-\frac{1}{2}} (t_{\text{inj}} - t_w) \quad (L_{\text{path}} \geq L_b) \]  \tag{20}

were obtained as the equations of path penetration.

Now, the spray path length from impingement point \( L_{\text{path}} \) and the elapsed time from impingement \( t_{\text{inj}} \) were defined as follows.

\[ L_{\text{path}} = L_{\text{path}} - L_w \]  \tag{21}

\[ t_{\text{inj}} = t_{\text{inj}} - t_w \]  \tag{22}

Then, the path length of post-impingement spray during the uni-velocity region was simply expressed as follows.

\[ L_{\text{path}} = A_{\text{pre}} t_{\text{inj}} \quad (L_{\text{path}} < L_b) \]  \tag{23}

\[ L_{\text{path}} = \frac{1}{2} A_{\text{free}} \left( t_w \right)^{-\frac{1}{2}} t_{\text{inj}} \quad (L_{\text{path}} \geq L_b) \]  \tag{24}

**Modeling of post-impingement spray**
The empirical formula eq.(7) of post-breakup spray explained that a spray was decelerated after the breakup process on account of the momentum exchange between injected fuel and surroundings, that is, the conservation of momentum. The growth behavior of post-uniform region should also obey this conservation of momentum. The origins of length and time were modified by the imaginary origin set on an impingement point. The modified spray path length $\tilde{\bar{L}}_{\text{path}}$ was considered to increase in proportion to the root of the modified time $\tilde{\bar{t}}_{\text{inj}}$ like radial jet\(^{12}\) and wall jet\(^{13}\). Then, $\tilde{\bar{L}}_{\text{path}}$ was expressed as follows.

$$\tilde{\bar{L}}_{\text{path}} = A' \left(\tilde{\bar{t}}_{\text{inj}}\right)^{\frac{1}{2}} \quad (\tilde{\bar{L}}_{\text{path}} \geq \tilde{\bar{L}}_{\text{uni}})$$ \quad (25)

Here, $\tilde{\bar{L}}_{\text{uni}}$ showed the length of uniform region from an impingement point. This equation showed that the coefficient $A'$ could be determined from the cross point of the two penetration lines shown in Fig.7.

The shift from eq.(23) to eq.(25) or from eq.(24) to eq.(25) occurred at $\tilde{\bar{L}}_{\text{path}} = \tilde{\bar{L}}_{\text{uni}}$ and $\tilde{\bar{t}}_{\text{inj}} = \tilde{\bar{t}}_{\text{uni}}$. Then, the value of $\tilde{\bar{t}}_{\text{uni}}$ and $\tilde{\bar{t}}_{\text{uni}}$ could be derived by coupling these two equations. When the impingement distance was shorter than the breakup length, $\tilde{\bar{L}}_{\text{uni}}$ and $\tilde{\bar{t}}_{\text{uni}}$ were obtained from eqs.(23) and (24).

$$\tilde{\bar{t}}_{\text{uni}} = \frac{A'^2}{A_{\text{pre}}} \quad (L_w < L_3)$$ \quad (26)

$$\tilde{\bar{L}}_{\text{uni}} = V_{\text{uni}} \tilde{\bar{t}}_{\text{uni}} = \frac{A'^2}{A_{\text{pre}}} \quad (L_w < L_3)$$ \quad (27)

For the long distance impingement spray ($L_w \geq L_3$), they were obtained from eqs.(24) and (25).

$$\tilde{\bar{t}}_{\text{uni}} = \frac{4A'^2L_w}{A_{\text{free}}^2} \quad (L_w \geq L_3)$$ \quad (28)

$$\tilde{\bar{L}}_{\text{uni}} = V_{\text{uni}} \tilde{\bar{t}}_{\text{uni}} = \frac{2A'^2}{A_{\text{free}}} \tilde{t}_{\text{inj}} \quad (L_w \geq L_3)$$ \quad (29)

The growth model for short distance impingement spray ($L_w < L_3$) is shown in Fig.8 and for the long distance impingement ($L_w \geq L_3$) is in Fig.9. The thick lines indicate the spray path length. The process of spray development was divided into four stages according to the state of mixture formation process. (I) Spray grew at uniform velocity before the break-up. (II) Spray developed as same as a free spray before impingement. In case of short distance impingement, this stage was skipped. (III) In the third stage, spray grew at uniform velocity because of the inactive exchange of momentum between spray and surroundings. (IV) And a spray grew in proportion to the root of the time from impingement after the uni-velocity stage.

By using these spray modeling, the spray growth behavior could be expressed as a set of simple formulas mentioned before.

**Empirical formula of impingement spray**

The coefficient $A_{\text{pre}}$ and $A_{\text{free}}$ were determined from experimental results of free spray. In this report, all the data were obtained using the nozzle, where the nozzle diameter was 0.24 mm, length-to-diameter ratio of the nozzle was 2.5 and opening pressure of the nozzle was 19.3 MPa. Using the penetration results of the free spray and eqs.(2) and (3), coefficient set of the penetration was determined as $A_{\text{pre}} = 114.1$ and $A_{\text{free}} = 57.0$.

The coefficient $A'$ was also determined from experimental data shown in Fig.10. A fitted line of square root function meant $\tilde{\bar{L}}_{\text{path}}$ in eq.(25). So the coefficient $A'$ could be determined by the value of $\tilde{\bar{L}}_{\text{path}}$ at $\tilde{\bar{t}}_{\text{inj}} = 1.0$ ms in Fig.10. In this case ($L_w = 10$ mm and $\phi = 90$ deg), $A'$ was obtained as 19.8.

Coefficient $A'$ was changed according to the wall condition such as $L_w$ and $\phi$. Then, we tested many
combinations of $L_w$ and $\phi$ to obtain the general relationship between $A'$ and wall parameters. After analyzing a lot of data, it was found that $A'$ could be expressed by the following functions of $L_w$ and $\phi$:

$$A' = F_1L_w + F_2\phi + F_3L_w\phi + F_4$$  \hspace{1cm} (30)

For the results concerning the test nozzle, this function was decided as follows:

$$A' = -0.36L_w - 0.39\phi + 0.0033L_w\phi + 56.1$$  \hspace{1cm} (31)

Comparison of the empirical formula and the measurements is shown in Fig.11. In this case, $A'$ was 42.4 which was obtained from eq.(31) substituting 10 mm for $L_w$ and 30 deg for $\phi$. The empirical formula showed well agreement with the measurements.

CONCLUSIONS

1) There was a stratified flow region of a high density fuel spray near the impingement point.
2) Spray grew with uniform velocity just after impingement. In this region, it was considered that the mixture formation process between an injected fuel and surroundings was limited. After the univelocity region, spray grew in proportion to the root of the elapsed time from impingement.
3) A spray growth model with uniform velocity region was newly introduced.
4) Penetration of impingement spray was expressed with an empirical formula as a function of impingement distance and impingement angle. The empirical formula showed well agreement with the measurements.

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