Static Analysis of Viscoelastic Supported Gas Foil Thrust Bearing with Journal Inclination*

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Abstract
Thrust pads in rotary machines are often used to bear the axial load, sometimes part or whole of the rotor weight is also sustained by thrust pads when the machine is not installed horizontally. Static characteristics of gas foil thrust bearing such as bearing load and friction torque play important roles in bearing-rotor system. In this paper, Finite Difference Method (FDM) was adapted to analyze compliant surface thrust gas bearing with viscoelastic support. Considering inclination of the journal, pressure distribution and film thickness between thrust disk and thrust pads were obtained by solving dimensionless governing equations, then nondimensional bearing load and nondimensional friction torque can be calculated subsequently. Based on the results of numerical calculation, effects of structural parameters and operational parameters on static characteristics were discussed. Theoretic analysis denote that: thrust foil bearing in journal inclination have higher load capacity and higher friction torque than that in no tilting condition; the optimum structure parameters for viscoelastic supported foil thrust bearing with the outer radius to inner radius ratio of 3 and dimensionless compliant coefficient of l is $\beta = 90^\circ$ and $b = 0.5$. Experiment about bearing load also has been implemented in this paper, and the results prove that the theoretical analysis is available.

Key words: Foil Bearings, Journal Inclination, Load Capacity, Friction Torque, Static Characteristics

1. Introduction
As supporting elements of rotary machines, foil bearings generally use air as lubricant. Due to the small viscosity and wide working temperature range of air, foil bearings offer many advantages such as longer work life and lower power consuming overall traditional bearings that are limited at extreme temperature or high speed. Moreover, foil bearings with compliant surface are not sensitive to thermal expansion and foreign particles, therefore the bearings show high stability in various working conditions.

With advantages mentioned above, many studies have been proposed and developed on foil bearings. However, comparing with the studies on foil journal bearings, studies on foil thrust bearings have been rarely reported. Etsion introduced a gas thrust bearing with cantilever mounted resilient thrust pads, and stiffness of the thrust bearing was discussed (1). Heshmat et al. obtained optimized structure parameters of bump foil thrust bearing by solving Reynolds equation with Finite Difference Method (FDM). They also compared the difference of static characteristics between stiff bearings and soft bearings (2). Iordanoff
proposed a rapid design method for foil thrust bearing, this method adapted simplified structural stiffness modeling and can save computing time obviously \(^{(3)}\). Using finite element method (FEM), Heshmat et al. calculated the deformation and displacement of compliant surface for bump foil thrust bearing; meanwhile they analyzed the hydrodynamic problem by FDM \(^{(4)}\). Considering tilting state and the rarefaction gas coefficient, Park et al. calculated static characteristics of bump foil thrust bearings; they also obtained dynamic characteristics by using perturbation method \(^{(5)}\). China began the studies on gas foil bearings for high speed turbines in 1980s, and Xi’an Jiaotong University proposed two kinds of foil bearings, namely, copper silk supported foil bearing and viscoelastic supported foil bearing by Xiong and Hou, respectively. On this basis, structural stiffness and running performance of viscoelastic supported foil thrust bearing were gained from experimental study \(^{(6-10)}\).

This paper is aim to analyze the effects of structural parameters and operational parameters on static characteristics of viscoelastic supported foil thrust bearing. As we known, in foil thrust bearing, pressure distribution and film thickness affect each other. When rotor is misalignment or distortion, journal becomes inclination with thrust bearing, and then film thickness will be changed directly. Thus, journal with various tilt angles were taken into consideration in the process of analysis. The results of this paper are benefit for producing viscoelastic supported foil thrust bearings with better performance.

2. Bearing structure and governing equations

As seen in Fig. 1 \(^{(10)}\), this foil thrust bearing is composed of some tiles with the same geometrical structure, and surfaces of the tiles are preformed into an inclined plane followed by a plat plane that is parallel to the thrust disk, which can form a convergent structure with the thrust face. Foil element of the thrust bearing consists of a smooth metallic top foil and a viscoelastic bottom foil, which is made of rubber. Top foil is supported by bottom foil that provides stiffness and damping for the bearing. One end of foil element is fixed on tile, and another end is free. When the rotor is running, gas is dragged into the convergent gap and pressure of gas increases by the hydrodynamic effect. This pressure rising can sustain the axial load generated by the pressure difference between two working wheels of the rotor.

![Fig. 1 Configuration of foil thrust bearing](image)

2.1 Reynolds Equation

In order to simplify the problem and deduce the governing equations of this foil thrust bearing, some assumptions are made as follows:
1) Top thin foil with smooth surface is flexible for thrust pad. Membrane effect and bending effect of top foil are ignored.

2) Stiffness of bottom foil is uniform and constant throughout the whole bearing surface.

3) The fluid in the gap of thrust bearing is steady and regarded as ideal gas.

Based on the configuration of foil thrust bearing and the above assumptions, the two-dimensional compressible Reynolds equation can be written as:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r h \frac{\partial p}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( r h \frac{\partial p}{\partial \theta} \right) = 6 \mu \omega \frac{\partial}{\partial \theta} \left( r h \right) \]

(1)

Where \( h \) is gas film thickness between bearing and thrust disk. How to analyze the film thickness is a key point for solving models of foil bearings. When bearing-rotor system is ideal assembled, film thickness of foil thrust bearing is just modified by gas pressure, so

\[ h = h_2 + g(r, \theta) + u \]

(2)

Where

\[ g(r, \theta) = \begin{cases} (h_i - h_2) \left( 1 - \frac{\theta}{b\beta} \right) & 0 < \theta < b\beta \\ 0 & b\beta \leq \theta \leq \beta \end{cases} \]

\( h_i \) and \( h_2 \) are inlet and outlet film thickness, respectively. \( u \) is the deformation of foil element. Under pressure \( p \), the deformation obeys the following equation:

\[ p - p_a - K u - B u = 0 \]

Where \( p_a \) is ambient pressure, \( K \) is structural stiffness, \( B \) is structural damping. The time term could be neglected in steady flow, so

\[ u = (p - p_a) / K = \alpha (p - p_a) \]

Where \( \alpha \) is defined as compliance coefficient of this foil thrust bearing. The higher value of \( \alpha \), the softer of the bearing.

The boundary conditions are

\[ \begin{cases} p = p_a & r = r_1 \quad \text{or} \quad r = r_2 \\ p = p_a & \theta = 0 \quad \text{or} \quad \theta = \beta \end{cases} \]

(3)

2.2 Influence of Journal Inclination

Rotors in practical machines are often misalignment or distortion, which is presented in Fig. 2 and Fig. 3. \( \varphi \) is the tilt angle of \( X \) direction, \( \psi \) is the tilt angle of \( Y \) direction, respectively. Because magnitudes of tilt angles are very small, the following relationships can be accessed

\[ \varphi \approx \sin \varphi, \quad \psi \approx \sin \psi \]

Thus under journal inclination, film thickness can be stated as follows:
2.3 Dimensionless Process

Setting \( \delta = h_1 - h_2 \) and using the following expressions to normalize governing equations

\[
\bar{p} = p / p_e, \quad \bar{r} = r / r_2, \quad \bar{h} = h / \delta, \quad \bar{g} = g / \delta, \quad \Lambda = (6 \mu \omega r_2^3) / (p_s \delta^2),
\]

\[
\alpha^* = p_e / k \delta, \quad \psi^* = \frac{r_2}{\delta} \psi, \quad \phi^* = \frac{r_2}{\delta} \phi.
\]

The dimensionless governing equations including Reynolds equation with relevant film thickness and boundary conditions become

\[
\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left( \frac{\bar{r}^2}{\Lambda} \frac{\partial \bar{p}}{\partial \bar{r}} + \frac{\partial}{\partial \theta} \left( \frac{\bar{r}^3}{\Lambda} \frac{\partial \bar{p}}{\partial \theta} \right) \right) = \Lambda \frac{\partial}{\partial \theta} \left( \bar{p} \bar{h} \right)
\]

\[
\bar{h} = \bar{h}_2 + \bar{g}(r, \theta) - \psi^* \bar{r} \cos \theta - \phi^* \bar{r} \sin \theta + \alpha^*(\bar{p} - 1)
\]

\[
\bar{g}(r, \theta) = \begin{cases} 
1 - \frac{\theta}{b \beta} & 0 < \theta < b \beta \\
0 & b \beta \leq \theta \leq \beta
\end{cases}
\]

\[
\begin{aligned}
\bar{p} &= 1 & \bar{r} &= \bar{r}_1 \quad \text{or} \quad \bar{r} = 1 \\
\bar{p} &= 1 & \theta &= 0 \quad \text{or} \quad \theta = \beta
\end{aligned}
\]

2.4 Local Coordinate System

In order to convenient for the solution of above Reynolds equation, local coordinate system is setup for every thrust pads, as seen in Fig. 4. First, pressure field of one pad is calculated, and then the pressure field of other pads will be obtained similarly by considering the relative position in thrust bearing. So, in the calculation, \( \theta \) of each pad has the same range which is between 0 and \( \beta \), but tilt angles have different ranges, given as follows:

\[
\begin{bmatrix}
\varphi_x \\
\varphi_y
\end{bmatrix} = \begin{bmatrix}
\cos \alpha_i & -\sin \alpha_i \\
\sin \alpha_i & \cos \alpha_i
\end{bmatrix} \begin{bmatrix}
\varphi_x \\
\varphi_y
\end{bmatrix}
\]
3. Analytical methods

After solving Reynolds equation of foil thrust bearing, pressure field and film thickness field can be obtained directly, and then these parameters can be integrated into dimensionless load capacity and dimensionless friction torque. These static characteristics can be expressed as:

\[
\bar{W} = \frac{W}{(p_r r^2)} = \int \left( \overline{p} - 1 \right) \overline{r} d\overline{r} d\theta, \quad W^* = \sum_{i=1}^{h} \bar{W}
\]

\[
\bar{T} = \frac{T}{(p_r \delta r^2)} = \int \left( \overline{h} r \frac{\partial \overline{p}}{\partial \theta} + \frac{\Lambda}{6} \frac{\partial}{\partial \theta} \right) \overline{r} d\overline{r} d\theta, \quad T^* = \sum_{i=1}^{h} \bar{T}
\]

For dividing the solution region into a uniform mesh, conformal mapping method was adapted to convert the bearing sector into a rectangular shape, as shown in Fig. 5. Setting \( F = \ln r \), Reynolds equation can be transferred to

\[
\frac{1}{\nu r^2} \frac{\partial}{\partial F} \left( \frac{h}{r} \frac{\partial \overline{p}}{\partial F} \right) + \frac{1}{\nu r^2} \frac{\partial}{\partial \theta} \left( \frac{h}{\partial r} \frac{\partial \overline{p}}{\partial \theta} \right) = \Lambda \frac{\partial (\overline{p} \overline{h})}{\partial \theta}
\]

The difference equation on point \((i, j)\) can be represented as

\[
f_{i,j}(\overline{p}_{i-1,j}, \overline{p}_{i,j}, \overline{p}_{i+1,j}, \overline{p}_{i,j-1}, \overline{p}_{i,j+1}) = 0
\]

Where \(i\) denotes radial direction and \(j\) denotes circumferential direction.

Dividing the radial direction into \(n\) grids and the circumferential direction into \(m\) grids, these nonlinear equations on all points were calculated until the pressure field and film thickness field had become converge by using direct iterative method.

4. Results

Considering the actual geometrical structure, default parameters for the foil thrust bearing were set as follows

\[
r_2 / r_1 = 3, \quad \beta = 90^\circ, \quad h_1 / h_2 = 3, \quad b = 0.5, \quad \Lambda = 1, \quad \alpha^* = 1, \quad \phi^* = 0, \quad \psi^* = 0, \quad n=30, \quad m=40
\]
After several iterations, profiles of pressure and film thickness for rigid surface bearing and compliant surface bearing were obtained. Figure 6 shows the three-dimensional pressure distribution of rigid surface bearing (\( \alpha^* = 0 \)), and Fig. 7 shows the corresponding surface shape. There is no deformation in rigid surface bearing, so this surface shape is the same as original one. Pressure distribution and corresponding surface shape of compliant surface bearing are presented in Fig. 8 and Fig. 9. It is found that the compliant surface was deflected obviously by film pressure, and the film pressure was affected by surface’s deformation vice versa. Under this coupled relationship, pressure distribution in compliant surface bearing becomes relatively flat.

Since thrust bearings have symmetric structure, tilt angles in two directions have the similar effect on static characteristics of thrust bearings, thus subsequent results just show the effect of \( X \) direction’s tilt angle.

Effects of tilt angle span proportion of ramp \( b \) are given in Fig. 10 and Fig. 11. In all cases, foil thrust bearings have higher load capacity when ramp \( b \) is between 0.4 and 0.6, and the load capacity of different tilt angles get closer when ramp \( b \) is bigger than 0.6. The relationship between friction torque and ramp \( b \) is about linear, and foil thrust bearings has higher friction torque under lower value of ramp \( b \). As presented in Fig. 12 and Fig. 13, load capacity and friction torque under all cases increase with the increase in bearing number \( \Lambda \).

In a fixed geometrical structure, bearing number reflects the change of rotation speed. According to theoretical results, it can be said that as rotation speed increases, bearing load of foil thrust bearing increases. Figure 14 and Fig. 15 show the relationship between static performances and thrust pad’s angle under various tilting conditions. It is seen that load capacity of all cases reach the peak value at about \( \beta = 90^\circ \). When thrust pad’s angle is less than 90°, load capacity gets rapid growth with the increase in pad’s angle; when the pad’s angle is larger than 90°, load capacity gets slow decline with the increase in pad’s angle. Friction torque on all tilting conditions decrease with the increase in thrust pad’s angle, in addition, the change of friction torque in small pad’s angle is quicker than that in wide pad’s.
At the same time, some experiments about viscoelastic supported foil thrust bearing have been conducted on a test rig which includes two parts: rotation part and loading part, as seen in Fig. 16 (10). The rotation part is supported by aerostatic bearings, and droved by an impeller. Rotor in rotation part can be accelerated to 120,000 rpm. In the loading part, a piston rod which can provide axial load by compressed gas is supported by aerostatic journal bearing, and test thrust bearing with 36 mm o.d, 12 mm i.d is fixed on the rod to sustain the rotor. Foil thickness of tested thrust bearing is 0.46 mm, including metallic top foil with 0.06 mm and viscoelatic foil with 0.4 mm. Bottom foil is made of fluorine rubber which can endure certain temperature and provide adequate stiffness and damping, and beryllium bronze that has favorable self-lubricating property, is processed into top foil. As presented in Fig.17, three eddy current displacement sensors are installed on the test rig for monitoring the status of the operating equipment. Two of them monitor radial motion of revolving rotor, and the rest monitors axial motion. The rotation speed of rotor is acquired by using FFT (Fast Fourier transform) method to analyze the resonance frequency of
vibration. It’s hard to get the height of actual film thickness and the change of actual film thickness can be obtained by comparing initial signal data and operating signal data of axial displacement sensor. Pressure of compressed gas is measured by pressure sensor or precision pressure gauge, and then load capacity can be easily calculated using pressure of compressed gas multiplied by sectional area of piston rod.

In order to facilitate the analysis between simulation and experimental results, actual bearing load has been converted into nondimensional value. The convert process is on the basis of Eq. (8), namely, nondimensional bearing load can be expressed as actual bearing load divided by the product of ambient pressure and square of bearing outer radius. Figure 18, which combines simulation and experimental results, shows the relationship of bearing load and ramp b. It is found that test curve is consistent with the theoretical curve, while bearing load in experiment is a bit bigger than that in simulation, maybe due to the higher structure stiffness of test bearing. As shown in Fig. 19, bearing load in experiment has higher value with higher rotation speed and this trend of experimental data agrees well with theoretical analysis.

When loaded, with no tilting condition, film thickness field in the parallel portion of thrust pads is nearly constant, so the height of this film thickness can be used to predict the load capacity of foil thrust bearings; with tilting condition, film thickness field in the parallel portion of thrust pads is not constant and is dependent on the local position, so the parameter is not convenient for evaluating the performance of foil thrust bearings. In present paper, tilt angle is adapted to predict the load capacity.

All graphs of theoretic analysis show that, under the same structural parameters and operational parameters, viscoelastic supported thrust bearing with large tilt angle has higher load capacity than that with small tilt angle or no tilting condition. Heshmat et al. and Iordanoff mentioned that gas thrust bearing has higher load capacity with smaller nominal film thickness. When the possible lowest minimum film thickness occurs, the maximum load capacity is obtained. Considering tilting condition of thrust pads, Park et al. has calculated the elastoaerodynamic problem of bump foil thrust bearing, and the analysis shows that load capacity of bump thrust bearing increases in the decease of film thickness.
and in the increase of tilt angle (5). It can be deduced from Eq. (4) that the minimum film thickness with tilting condition is smaller than that with no tilting condition, and the load capacity increases with the decreasing of minimum film thickness which has been proved in Refs. (2) and (3), so test thrust bearing with tilting condition has larger load capacity, and this trend is in accordance with above studies. It must be pointed out that the tilt angle adapted in present paper is rather small, considering that the lubricating film will break when tilt angle exceeds certain value.

5. Conclusions
In this paper, the static characteristics of foil thrust bearing with viscoelastic supported were discussed. Considering journal inclination, governing equations of foil thrust bearing, including Reynolds equation and relevant expressions of film thickness and boundary conditions, were proposed by using some assumptions which mainly included that: (1) the stiffness of bottom foil is constant and uniformly distributed; (2) the top foil just provides smooth flexible surface for the thrust pads. Then governing equations were solved by FDM using uniform meshes which have 30 grids in radial direction and 40 grids in circumferential direction, respectively. By intergrading the resulting pressure distribution and film thickness, the relationships of static characteristics and relevant parameters under different tilt angles were obtained.

Journal in tilting condition is benefit for foil thrust bearing to carry higher load, at the same time, the bearing sustains higher friction torque. As rotation speed increases, bearing load and friction torque of foil thrust bearing increase. Load capacities have peak value in certain range of ramp b and thrust pads’ angle, meanwhile friction torques decrease with the increase of the two parameters. Thus, the optimum structural parameters for viscoelastic supported foil thrust bearing with the outer radius to inner radius ratio of 3 and dimensionless compliant coefficient of 1 is  β = 90° and  b = 0.5.

Theoretic results agree well with relevant experimental data, so the model of viscoelastic supported foil thrust bearing proposed in the paper can be regarded as credible. These simulation results provide guidance for the structure design of foil thrust bearing with viscoelastic support. Combining with further experiment study, this bearing is expected to use in practical high speed rotary machines.

6. Discussions
Heshmat et al. (1983), Iordanoff (1997) and Park et al. (2008) have carried out analytical studies on bump foil thrust bearing. Following the solution of the governing equation, the geometry of bump foil thrust bearing was optimized by Heshmat et al (2). Based on the results obtained from rigid profiles, the optimal initial profile of bump foil thrust bearing was rapidly determined by Iordanoff. This work not only reduces the calculation time but also greatly improves load capacity (3). Considering tilt and slip flow, load capacity of bump foil thrust bearing has been analyzed by Park et al., moreover, static characteristics and dynamic characteristics have been calculated (5). Comparing to above studies on bump foil thrust bearing, the present paper concerns on solving the elastoaerodynamic problem of viscoelastic supported foil thrust bearing and exploring the effects of structural and operational parameters on bearing performances. Besides, journal inclination is a key consideration in this paper. Park et al. regarded the eccentricity of rotation journal as the reason of tilting condition of thrust pads, and just simply presented the performances of bump foil thrust bearing in tilting condition under various bearing number. In contrast, present paper regarded rotor’s distortion or misalignment as the reason of journal inclination and film thickness will be direct changed in tilting condition, thus the
performance of thrust bearing is heavily affected by journal inclination. Under various structural and operational parameters, performances of viscoelastic supported foil thrust bearing with various tilt angles are described detailedly in the paper.

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**Nomenclature**

- $b$: extent of ramp as fraction of pads’ angle
- $h$: film thickness (m)
- $h_1$: inlet film thickness (m)
- $h_2$: outlet film thickness (m)
- $\bar{h}$: dimensionless film thickness (m)
- $\bar{h}_1$: dimensionless inlet film thickness
- $\bar{h}_2$: dimensionless outlet film thickness
- $i$: node number in radial direction
- $j$: node number in circumferential direction
- $k$: pad number
- $p$: pressure (N/m$^2$)
- $p_a$: ambient pressure (N/m$^2$)
- $\bar{p}$: dimensionless pressure
- $r_1$: bearing inner radius (m)
- $\bar{r}_1$: dimensionless inner radius
- $r_2$: bearing outer radius (m)
- $\bar{r}_2$: dimensionless outer radius
- $r, \theta$: cylindrical coordinate
- $F$: function of coordinate transformation
- $W$: load capacity of single pad (N)
- $\bar{W}$: dimensionless load capacity of single pad
- $W^*$: total dimensionless load capacity
- $\bar{T}$: friction torque of single pad (N.m)
- $\bar{T}^*$: total dimensionless friction torque
- $\alpha$: compliance coefficient
- $\alpha^*$: dimensionless compliance coefficient
- $\beta$: angular extent of pad
- $\delta$: difference of inlet film thickness and outlet film thickness (m)
- $\mu$: viscosity of air (Pa.s)
- $\omega$: angular velocity (rad/s)
- $\Lambda$: bearing number

**References**

(1) Etsion, I., Cantilever mounted resilient pad gas thrust bearing, Transaction of the ASME,


