Simultaneous Five-axis Motion for Identifying Geometric Deviations Through Simulation in Machining Centers with a Double Pivot Head

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Abstract
In this paper, simultaneous five-axis control motions were newly proposed to identify the ten inherent deviations to double pivot head type five-axis machines. This motion was designed for the application of a ball bar to the measurements. The ball bar can be applied to the motion by changing the ball bar’s sensitive direction in the axial, radial and tangential directions in relation to the A-axis rotation. All the three ball bar settings can be changed in a single arrangement, with a magnetic clamp fixed onto the main spindle. This leads to reduce the setting errors and setting time as well. The ten deviations are identified by using three measurements, two of which are taken in the axial direction motion while the third is extracted using the tangential or radial direction motion. From the axial direction motion, seven deviations are estimated; four of these estimations are made using the observation equation while the other three are made using simple geometric relations. The remaining three of the ten deviations are determined by using tangential or radial motion. The validity of the proposed method is confirmed by simulations.

Key words: Machining Center Five-Axis Motion, Double Pivot Head, Geometric Deviation, Identification, Simulation, Ball Bar

1. Introduction

Five-axis machining centers with a double pivot head have been used for manufacturing aero-components, dies and punches for automobile, propellers for ship, and impellers for compressors and so on. The oldest five axis machining center in the history was the double pivot head type(1). NAS 979 standard(2) was developed in 1969 for checking the machining accuracy of a cone frustum. Its test piece must be finished by simultaneous five axis motion. In 1998, the test standards for checking the geometric deviations inherent to the double pivot head type were specified by ISO (3)-(5). A kinematic test for the double pivot head type was also specified in the ISO standards(6). ASME also described several testing methods in the annex(7).

In Annex B of ISO 10791-1(8), seven objects for inspecting the accuracy level by using basic instruments such as straightedges, squares, precision levels and dial gages. Seven setups for the measurement instruments must be conducted. It is, however, inconvenient and time consuming that in those standards, at every time the setup has to be changed to the relevant positions according to the purpose of the test.

In addition, there are two test standards relating to the kinematic test. One is the clause...
K6 of ISO 10791-6\textsuperscript{(6)} which is specified to check the spherical interpolation motion. However, it is already pointed out that it is difficult to apply this motion to the identification of the geometric deviations inherent to the double pivot head type\textsuperscript{(3)}. The other is Annex F of the ASME B5.54\textsuperscript{(7)} which illustrates seven ball bar patterns on four- and five-axis machines in the figures. Any those patterns have not been examined and also any simultaneous five-axis motions have not been proposed.

Many research works have been carried for measuring the static and kinematic behaviors of five-axis machining centers with a tilting rotary table\textsuperscript{(9)-(19)}. It is well known that in the five-axis machining centers the number of the geometric deviations inherent to the tilting rotary table type is less than that inherent to the double pivot type. Therefore, it is difficult to estimate the individual deviations inherent to the double pivot type as compared with the tilting rotary table type.

Tsutsumi and Saito\textsuperscript{(18), (19)} already reported two calibration methods for five-axis machines with a tilting rotary table by means of a ball bar. Especially, they proposed a simultaneous four-axis motion to estimate the geometric deviations\textsuperscript{(19)}. It is, however, not easy to apply the simultaneous four-axis motion to the double pivot type. Any motion tests are not conducted to inspect the geometric accuracy of the double pivot head type except our reports\textsuperscript{(8)}, in which a strategy for identifying the geometric deviations of the double pivot head type are discussed. This method is basically based on the clause K6 of ISO 10791-6 and the simultaneous three axis motion was applied to the measurement. The methodology required four measurements to identify the deviations and also the setting of the jigs which were used to achieve the motions were not easy and it was a time consuming work. If a new methodology can be proposed with less number of measurements and simple jigs, that is very useful for the manufactures as well as machine shops as an acceptance test.

The five-axis simultaneous control motion which proposed in this paper has not been used by any researcher to identify the geometric deviations inherent to five-axis machining centers with a double pivot head. In this motion all axes are involved to the motion at the same time and thus all or most of the geometric deviations affect to the motion trajectory. Therefore, the potentiality of the motion to use for identifying the deviations is very high.

In this paper, three simultaneous five-axis motions were proposed and identification methodology for the deviations inherent to double pivot head type machining centers is discussed. This methodology consists of three steps and all the measurements were taken by means of ball bar and used only one jig to setup the ball bar for the measurements.

In the first step, four of ten deviations were identified by using the A-axis axial motion in which the ball bar is applied parallel to the axis of the A axis motion. Three of the remaining deviations are identified in the second step by using two measurements of the A-axis axial motion and by considering the geometry of the machine. Remaining deviations are identified in the third step by using either A-axis tangential motion or radial motion in which the ball bar is applied in tangential or in radial to the A axis motion, respectively. Validity of the proposed methodology for identifying the deviations inherent to double pivot head type five-axis machining centers is confirmed by the simulations.

2. Inherent Deviations and Simulation Method

2.1 Inherent deviations to double pivot head type

Figure 1 shows an example of the double pivot head type five-axis machining center with a horizontal Z-axis\textsuperscript{(3)}. This type of machines having all controlled axes in the spindle head side while the table is fixed is more common in the horizontal spindle type as shown in the reference\textsuperscript{(19)}. The spindle head consists of two rotary axes and therefore it is called as a double pivot head. This is also called as a
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Among the ten deviations, three deviations $\alpha_{SA}$, $\gamma_{CZ}$, and $\delta_{SA}$ are adjustable. The large suffixes indicate the related axes while the small suffixes indicate the direction of the deviation. However, when we think about the meaning of the deviation $\beta_{CZ}$ described by the suffixes, $\beta_{CZ}$ is in the joint between C and Z axes around Y coordinate frame. Therefore, it can be said that the $\beta_{CZ}$ represents the parallelism between the C-axis and Z-axis motion or squareness between the C-axis and X-axis motion ($\beta_{CX}$). That means $\beta_{CZ}$ can be written as $\alpha_{C}$, representing the squareness between the spindle axis and Y-axis motion and it can, therefore, be said that $\alpha_{C}$ is equal to $\alpha_{Y}$. These deviations are independent from each other.

Table 1 Definitions of inherent deviations to the five-axis machining center

<table>
<thead>
<tr>
<th>Deviations</th>
<th>Definition of the deviation</th>
<th>Tool/spindle and A-axis</th>
<th>A-axis and C-axis</th>
<th>C-axis and Z or X-axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{SA}$ (mm)</td>
<td>Difference between ideal and actual positions of spindle nose (adjustable)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_{SA}$ (mm)</td>
<td>Coincidence between spindle axis and A-axis center line in XY plane in Y direction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_{SA}$ (mm)</td>
<td>Coincidence between spindle axis and A-axis center line in XY plane in X direction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_{SA}$ (°)</td>
<td>Squareness between spindle axis center line Y-axis movement ($\alpha_{Y}$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{SA}$ (°)</td>
<td>Squareness between spindle axis center line and A-axis center line</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_{AC}$ (mm)</td>
<td>Offset between A-axis origin and C-axis center line in Y direction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{AC}$ (°)</td>
<td>Squareness between A-axis center line and C-axis center line</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_{CZ}$ (°)</td>
<td>Squareness between C-axis center line and Z-axis movement ($\alpha_{Z}$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{CZ}$ (°)</td>
<td>Squareness between C-axis center line and X-axis movement ($\beta_{X}$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_{CZ}$ (°)</td>
<td>Initial angular position of C-axis (adjustable)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.2. Simulation method

Figure 2 illustrates the ten inherent deviations which are defined in Table 1 and the coordinate frames that were used to establish the mathematical model for the simultaneous five-axis motion. In this simulation, it is supposed that a ball bar system is used to measure the relative displacement between the tool and the workpiece. The coordinates of the center of the ball on the spindle side according to the origins of the coordinate systems at the A-axis, C-axis, and machine are defined as $S_{SA}(X_{SA}, Y_{SA}, Z_{SA})$, $S_{SC}(X_{SC}, Y_{SC}, Z_{SC})$, and $S_{SM}(X_{SM}, Y_{SM}, Z_{SM})$, respectively. $R_a$ is the distance between the coordinate systems at the center of the ball on the spindle side and the center line of the A-axis.

By considering the connection between the origins of the coordinate systems at the center of the ball on the spindle side and the center line of the A-axis, $S_{SA}$ can be formulated as Eq. (1).

$$
\begin{bmatrix}
X_{SA} \\
Y_{SA} \\
Z_{SA}
\end{bmatrix}
= D_{SA}D_{aSA} \begin{bmatrix}
1 \\
1 \\
-1
\end{bmatrix}
\begin{bmatrix}
\delta_{xSA} \\
\delta_{ySA} \\
\delta_{zSA}
\end{bmatrix}
$$

(1)
where, $D$ denotes the effect of angular deviation for the motion.

$$D_a = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}$$  \hspace{1cm} (2)$$

$$D_\beta = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix}$$  \hspace{1cm} (3)$$

$$D_\gamma = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$  \hspace{1cm} (4)$$

By considering the motion of the coordinate system at the A-axis reference to the coordinate system at the C-axis the next relationship can be drawn.

$$\begin{bmatrix} X_{ac} \\ Y_{ac} \\ Z_{ac} \end{bmatrix} = D_{bac} D_{\beta} D_{\gamma} E_a \begin{bmatrix} X_{ac} \\ Y_{ac} \\ Z_{ac} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$  \hspace{1cm} (5)$$

where, $E_a$ denotes the rotation of the A-axis and can be expressed as below.

$$E_a = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix}$$  \hspace{1cm} (6)$$

From the relationship of the coordinate system at the C axis and the machine coordinate system, $S_{SM}(X_{SM}, Y_{SM}, Z_{SM})$ can be also expressed as given in Eq. (7).

$$\begin{bmatrix} S_{SM} \end{bmatrix} = D_{jc2} D_{jc2} D_{ac2} E_c \begin{bmatrix} X_{ac} \\ Y_{ac} \\ Z_{ac} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$  \hspace{1cm} (7)$$

where, $E_c$ is the rotation of the C-axis, and $X$, $Y$ and $Z$ are the moving distances of the $X$, $Y$ and $Z$ axes respectively.

$$E_c = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$  \hspace{1cm} (8)$$

As above, the coordinates of the center of the ball on the table side according to the machine coordinate system can be formulated as in Eq. (9).

$$\begin{bmatrix} X_{TM} \\ Y_{TM} \\ Z_{TM} \end{bmatrix} = D_{jc2} D_{jc2} D_{ac2} D_{bac} D_{bac} \begin{bmatrix} \delta_{a\delta} \\ \delta_{a\delta} \\ -R_a + \delta_{a\delta} \end{bmatrix} \begin{bmatrix} X_{ac} \\ Y_{ac} \\ Z_{ac} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ R_a - R_T \end{bmatrix}$$  \hspace{1cm} (9)$$

where, $R_T$ is the distance between the center of ball on table side and the origin of the coordinate system at A-axis.

Thus the deviation $e$ of any point on the trajectory can be expressed as in Eq. (10). This mathematical model is used to carry out the simulations.

$$e = \sqrt{(X_{SM} - X_{TM})^2 + (Y_{SM} - Y_{TM})^2 + (Z_{SM} - Z_{TM})^2 - L_B}$$  \hspace{1cm} (10)$$

where, $L_B$ is the length of ball bar.
3. Sensitive Direction of the Ball Bar Under Five-axis Motion

3.1. Ball bar setup and the motion

To measure the relative displacement between the tool and the workpiece by means of a ball bar instrument, the bar’s magnetic sockets are fixed on the spindle nose and the table through some jigs to avoid interference between the table and the spindle head, and the ball bar is then set between these magnetic sockets. In general, the magnetic socket on the spindle side is clamped by a collet chuck holder and the magnetic socket on the table side is magnetically clamped to the table.

In the simultaneous five-axis control motion, there are three sensitive directions in the measurement using the ball bar, as shown in Fig. 3. First is the axial direction, in which the sensitive direction of the ball bar is parallel to the center line of the A-axis. Second is the radial direction, in which the sensitive direction is perpendicular to the center line of the A-axis, as shown in Fig. 3(b). Third is the tangential direction, in which the sensitive direction is tangential to the rotation of the A-axis as shown in Fig. 3(c).

In the axial direction, the distance $r_2$ between the center of the A-axis and the center of the ball on the tool spindle was set to 400mm in the simulation. The ball on the table was fixed during the measurement cycle, and only the ball on the spindle was moved, according to the NC program. In this measurement, only the magnetic clamp is used as a jig, as the figure illustrates. To achieve the simultaneous five-axis motion, the C-axis rotates 0° to 180° and 180° to 360° in one cycle; the A-axis rotates 0° to 90° and 90° to 0°; the X, Y and Z axes are moved simultaneously so as to keep the ball bar parallel to the center line of the A-axis; and the distance between the center of the balls on the table and spindle is kept constant. By using the NC data of the X, Y, Z, A and C axes, we can apply the mathematical model shown in Eq.(10) to simulate the motion trajectory.

In the radial direction, the ball bar setup and the positions of the A and C axes are illustrated in Fig. 3(b). The ball bar is applied perpendicular to the center line of the A-axis. Throughout the motion, the X, Y and Z axes are moved to maintain a constant relationship between the ball bar and the A-axis. The C-axis rotates 0° to 180° and 180° to 360° in one cycle, while the A-axis rotates 0° to 90° and 90° to 0°.

In the tangential direction, the ball bar is set as shown in Fig. 3(c). The X, Y and Z axes are moved to keep the ball bar tangential to the rotation of the A-axis. Meanwhile, the A-axis is rotated 0° to 90° and 90° to 0° and the C-axis is rotated 0° to 180° and 180° to 360° respectively.

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![Fig. 3 Ball bar settings in three types of simultaneous five-axis control motions](image-url)
4. Identification of Deviation – Axial Direction

4.1. Identification of deviations by using observation equation

The reference data, those are the motion trajectories corresponding to the individual deviations, have to be prepared prior to the identification of the deviations by the observation equation (19). To investigate the patterns, simulations were carried out using the mathematical model described above. The ten deviations were applied to the model one by one, keeping the initial conditions unchanged. To extract the reference data, 0.01mm and 0.01° were given to the linear and angular deviations, respectively, for the simulations as same as the reference (19).

Figure 4 provides the simulated motion trajectories by applying each deviation to the motion. These all are plotted against to the axis rotation, which is 0° to 360° and named as one cycle. As illustrated in Fig.4(a), the δSA, which was identified as an adjustable deviation, does not affect to the axial direction motion trajectory. Thus it is possible to ignore the effect of δSA for this motion. The deviations αCZ, βCZ, γCZ and βAC show quite different behaviors throughout the cycle. Therefore, we can consider the each of these four patterns as unique.

As mentioned above, the δAC, δSA and αSA show sine curve behaviors, while as shown in Fig. 4(c), βSA and δSA show cosine curve behaviors. In both patterns, the linear deviation couples with the angular deviation. This is an interesting relationship, which can be used to separate the two deviations. Furthermore, δAC and δSA show identical behaviors, as shown in Fig.4 (b). These two behaviors, illustrated in Fig.4 (b) and (c), can be considered as two different patterns.

In general, the angular deviations show larger influences than the linear deviations. That is because the influences of the angular deviations depend directly on the related length of the component as Ra. It can also be seen that the peak reading (0.02mm) of the trajectory of δSA is twice the peak reading (0.01mm) of the trajectories of δSA and δAC. This is because the δSA is in the same direction as the sensitive direction of the ball bar and the ball bar direction reverses at 180° of the cycle, as shown in Fig. 4(c), which is the peak point of the trajectory of δSA.

As mentioned above, the

![Fig.4 Influence of deviations to the motion trajectory of simultaneous five-axis motion in axial direction](image)

<table>
<thead>
<tr>
<th>Deviation</th>
<th>Value</th>
<th>Deviation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>δSA (mm)</td>
<td>-0.0051</td>
<td>βSA (°)</td>
<td>0.0048</td>
</tr>
<tr>
<td>δSA (mm)</td>
<td>0.0073</td>
<td>βAC (°)</td>
<td>0.0039</td>
</tr>
<tr>
<td>δSA (mm)</td>
<td>-0.0180</td>
<td>αCZ (°)</td>
<td>-0.0065</td>
</tr>
<tr>
<td>δAC (mm)</td>
<td>-0.0077</td>
<td>βCZ (°)</td>
<td>-0.0084</td>
</tr>
<tr>
<td>αSA (°)</td>
<td>0.0031</td>
<td>γCZ (°)</td>
<td>0.0120</td>
</tr>
</tbody>
</table>
deviations $\beta_{AC}$, $\alpha_{CZ}$, $\beta_{CZ}$ and $\gamma_{CZ}$ have unique patterns, as shown in Fig. 4 (a), and the deviations $\alpha_{SA}$, $\delta_{SA}$ and $\delta_{AC}$ have also sine curve behavior as shown in Fig. 4(b), while the deviations $\beta_{SA}$ and $\delta_{SA}$ have cosine curve behavior, as shown in Fig. 4(c). As a whole, we can identify six different patterns, and these data can be considered as a master data set for the observation equation (19).

The measured data set can be extracted from the trajectory for $R_a=400$ mm given in Fig. 5. All the random values for the deviations as shown in Table 2 are input to the simulation to extract the trajectory given in Fig. 5. This trajectory is identical to the trajectory extracted from the ball bar. Therefore, we can consider these data as the data measured by the ball bar.

Table 3 shows the estimated results by means of the observation equation. According to the table, the deviations $\beta_{AC}$, $\alpha_{CZ}$, $\beta_{CZ}$ and $\gamma_{CZ}$ are accurately estimated by using the observation equation. Among these four deviations, $\gamma_{CZ}$ was identified as an adjustable deviation. Also all four identified deviations are angular deviations.

The readings of $\alpha_1$ and $\beta_1$ give the cumulative influences of linear and angular deviations in the two groups shown in Figs. 4 (b) and (c), respectively. In this case, among the unidentified deviations, two angular deviations and four linear deviations exist. Hence, $\delta_{SA}$ does not affect on the axial motion and can be neglect. Finally, the two categories given in Fig. 4 (b) and (c) remain. To separate these deviations, we can try to use the relationship between the angular and linear deviations in each group.

4.2. Identification of deviations by using geometric relations

The influences of angular deviations depend on the length of the component immediately related to the angular deviations.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Deviation & Given value & Estimated value \\
\hline
$\alpha_1$ & -0.03873 & - \\
$\beta_1$ & 0.02122 & - \\
$\alpha_2$ & -0.04716 & - \\
$\beta_2$ & 0.02663 & - \\
$\beta_{AC}$ & 0.0039 & 0.00388 & 0.00388 \\
$\alpha_{CZ}$ & -0.0065 & -0.00651 & -0.00651 \\
$\beta_{CZ}$ & -0.0084 & -0.00841 & -0.00841 \\
$\gamma_{CZ}$ & 0.0120 & 0.01200 & 0.01200 \\
\hline
\end{tabular}
\caption{Estimated deviations at $R_a=400$ mm and 500 mm}
\end{table}

Fig. 5 Influence of all random deviations on the motion trajectory in the axial direction ($R_a=400$ mm and 500 mm)

Fig. 6 Effect of $R_a$ on the influence of deviations on motion trajectory in the axial direction
such as the distance $R_a$, which can be changed very easily by introducing an extension bar between the magnetic socket and the spindle. To investigate the effect of $R_a$ on the motion trajectory, $R_a$ was changed to 500mm by introducing a 100 mm extension bar and simulations were then carried out. The influences of nine deviations (that is, all except $\delta_{SSA}$) on the motion trajectory are extracted. Figure 6 shows the influences of three linear deviations and one angular deviation, which represent all angular deviations.

It can be seen from the figure that the change in $R_a$ does not affect on the influences of the linear deviations, whereas it does affect the angular deviations. Furthermore it can be seen that all enlargements are equal to 25 % of the base curve figures at $R_a=400$ mm. That percentage is the same as the percentage of the length $R_a$ extension. This means there is a direct relationship between the length $R_a$ and the influences of angular deviations.

To evaluate the new cumulative effects of the two groups of deviations given in Figs.4 (b) and (c), the identification was carried out. These effects in Figs.4 (b) and (c) were renamed as $\beta_2$ and $\alpha_2$, respectively. To prepare the master data set, the data related to $R_a=500$ mm, shown in Fig.6 were used. The measured data set was extracted from the trajectory for $R_a=500$ mm shown in Fig.5, which is the cumulative influence of all the random values of deviations. Estimated deviations are given in Table 3.

According to Table 3, only the readings of $\alpha_1$ and $\beta_1$ are changed. That means these figures can be expressed by using geometric relations. Figure 7 illustrates the linkage between the coordinate frames at the spindle ($O_s$) and the A-axis ($O_A$) projected on the ZX plane. This expresses the relationship between $\alpha_1$ and $\alpha_2$. According to the figure, $\delta_{SSA}$ can be formulated by using $Ra_1$, $Ra_2$, $\alpha_1$ and $\alpha_2$ as below.

$$\delta_{SSA} = Ra_1 \sin \beta_{SSA} + \alpha_1$$ (11)

$$\delta_{SSA} = Ra_2 \sin \beta_{SSA} + \alpha_2$$ (12)

By manipulating Eqs.(11) and (12), Eq.(13) for $\beta_{SSA}$ can be derived.

$$\beta_{SSA} = \sin^{-1}\left(\frac{\alpha_2 - \alpha_1}{Ra_2 - Ra_1}\right)$$ (13)

Then, by substituting the value of $\sin\beta_{SSA}$ into Eq.(11) , the relationship for $\delta_{SSA}$ can be extracted as,

$$\delta_{SSA} = \frac{Ra_2\alpha_1 - Ra_1\alpha_2}{Ra_2 - Ra_1}$$ (14)

The linkage of the coordinate frames at the spindle ($O_s$), the A-axis ($O_A$), the C-axis ($O_C$) and the machine ($O_M$) projected to the YZ plane is given in Fig. 8. This expresses the relationship between $\beta_1$ and $\beta_2$. ($\delta_{SSA} + \delta_{AC}$) can be formulated by the aid of Fig. 8 as given
below.
\[
\delta_{\text{S4}} + \delta_{\text{SAC}} = \beta_1 - R_{\text{S1}} \sin \alpha_{\text{S4}} \quad (15)
\]
\[
\delta_{\beta4} + \delta_{\text{SAC}} = \beta_2 - R_{\text{S2}} \sin \alpha_{\text{S4}} \quad (16)
\]
By deducting Eq. (16) from Eq. (15) and carrying out some manipulations, we can write the relationship for \( \alpha_{\text{S4}} \) as:
\[
\alpha_{\text{S4}} = \sin^{-1}\left( \frac{\beta_1 - \beta_2}{R_{\text{S1}} - R_{\text{S2}}} \right) \quad (17)
\]
By substituting Eq. (17) into Eq. (15), the relationship for \((\delta_{\text{S4}} + \delta_{\text{SAC}})\) can be written as:
\[
\delta_{\text{S4}} + \delta_{\text{SAC}} = \frac{R_{\text{S1}} \beta_2 - R_{\text{S2}} \beta_1}{R_{\text{S1}} - R_{\text{S2}}} \quad (18)
\]
The values \(R_{\text{S1}}, R_{\text{S2}}, \alpha_1, \alpha_2, \beta_1 \) and \( \beta_2 \) are given in Table 3. By Eqs. (13), (14), (17) and (18), the values for \( \beta_{\text{S4}}, \delta_{\text{S4}}, \alpha_{\text{S4}} \) and \((\delta_{\text{S4}} + \delta_{\text{SAC}})\), can be estimated, respectively. The estimated values are given in Table 4. It can be seen that this simple geometry can accurately identify \( \delta_{\text{S4}}, \beta_{\text{S4}} \) and \( \alpha_{\text{S4}} \).

We have been able to accurately estimate seven deviations by using two measurements extracted by applying the simultaneous five-axis control motion in the axial direction at two different \( R_z \) values. However, this leaves three deviations \( \delta_{\beta4}, \delta_{\text{AC}} \) and \( \delta_{\text{S4}} \) those cannot be estimated by means of axial direction motion. Therefore, another two ball bar settings in the simultaneous five-axis control motion were investigated.

5. Identification of Deviations \( \delta_{\text{S4}}, \delta_{\text{AC}}, \delta_{\text{S4}}, \beta_{\text{S4}} \)

5.1. Radial direction
In the radial direction, four different patterns of effects on the motion trajectory can be examined, as illustrated in Fig. 9. Three deviations \( \delta_{\text{S4}}, \delta_{\text{AC}} \) and \( \delta_{\text{S4}} \) show the same pattern of behavior as given in Fig. 9(a), while four deviations \( \beta_{\text{S4}}, \beta_{\text{AC}}, \beta_{\text{C4}} \) and \( \delta_{\text{S4}} \) show full sin curve behavior, as illustrated in Fig. 9(b). \( \gamma_{\text{C4}} \) does not affect on the radial motion and can be neglected. The behaviors of \( \delta_{\beta4} \) and \( \delta_{\text{AC}} \) differ from those of the others as given in Fig. 9(c). However since seven deviations were identified in the above

<table>
<thead>
<tr>
<th>Deviation</th>
<th>Given value</th>
<th>Estimated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_{\text{S4}} ) (mm)</td>
<td>-0.0051</td>
<td>-0.00501</td>
</tr>
<tr>
<td>( \beta_{\text{S4}} )</td>
<td>0.0048</td>
<td>0.00483</td>
</tr>
<tr>
<td>( \delta_{\text{S4}} + \delta_{\text{AC}} ) (mm)</td>
<td>-</td>
<td>-0.00040</td>
</tr>
<tr>
<td>( \alpha_{\text{S4}} )</td>
<td>0.0031</td>
<td>0.00310</td>
</tr>
</tbody>
</table>
chapter by the axial direction motion, we can ignore all the identified deviations. Thus we can say that $\delta_{SA}$, $\delta_{AC}$ and $\delta_{SA}$ show unique patterns of behavior.

As in the above chapter, the observation equation was used to identify the three deviations. A master data set was extracted from the data illustrated in Fig. 9.

The measured data set was extracted by applying all the ten random deviations, shown in Table 2, to the model. Figure 10 shows the measured data set. However, according to the assumption that all the deviations affect on the motion trajectory linearly, the effects of the seven identified deviations can be deducted from the measured data set.

Thus, the cumulative effect of $\delta_{SA}$, $\delta_{AC}$ and $\delta_{SA}$ was extracted as given in Fig. 10. The identified values for the deviations by means of the observation equation are given in Table 5. It can be seen from the table that the three deviations $\delta_{SA}$, $\delta_{AC}$ and $\delta_{SA}$ can be identified accurately.

5.2. Tangential Direction

The ball bar is alternatively set in the tangential direction to identify the three deviations $\delta_{SA}$, $\delta_{AC}$ and $\delta_{SA}$. The ball bar setting for the motion is illustrated in Fig. 3(c). The X, Y and Z axes are moved to keep the ball bar tangential to A-axis motion. Meanwhile, the A-axis is rotated 0° to 90° and 90° to 0°, and the C axis is rotated 0° to 180° and 180° to 360° respectively.

As in the above section, all of the influences of deviations on motion were simulated. The results are given in Fig. 11. According to Fig. 11(a), $\gamma_{CZ}$ does not affect on the tangential direction motion and thus can be neglected. The other four deviations ($\delta_{AC}$, $\delta_{SA}$, $\alpha_{CZ}$ and $\beta_{CZ}$) have different patterns of influences on the motion trajectory. Among these four, only $\delta_{AC}$ and $\delta_{SA}$ are not yet identified.

The two deviations $\delta_{SA}$ and $\alpha_{SA}$ show the similar behaviors, as illustrated in Fig. 11(b). Among them $\alpha_{SA}$ was already identified, as given in Table 4. Therefore we can remove the effect of $\alpha_{SA}$ from the cumulative effect of $\delta_{SA}$ and $\alpha_{SA}$. Thus, we can consider the $\delta_{SA}$ has a unique influence on the motion trajectory.

Similar behaviors are exhibited by the deviations $\delta_{SA}$, $\beta_{SA}$ and $\beta_{AC}$ as illustrated in...
were already identified. From these data, the master data set was prepared. The measured data set included all the influences of the ten deviations is given in Fig. 12. However, as mentioned above, seven deviations already estimated. Based on the assumption that all the deviations influence to the motion trajectory in a linear manner, it is possible to deduct the influences of the seven deviations from the measured trajectory. The remaining trajectory is the cumulative influence of the three deviations that were not identified. This trajectory is also shown in Fig. 12. The measured data set for the calculations was extracted from the trajectory shown in Fig. 12.

The deviations were estimated by using the observation equation (19). Estimated results are given in Table 5. The table shows that the three deviations can also be identified by means of tangential direction motion. Therefore, we can estimate $\delta_{xS}$, $\beta_{xS}$ and $\beta_{AC}$ from any one motion (radial or tangential).

6. Comparison between the Proposed Method and the Other Methodology

A comparison of the two methods is instructive. As shown in Table 6, three measurements are required for identifying the deviations in the proposed method. This is fewer than the number of measurements in the methodology (8) related to ISO 10791-6. This means the proposed method reduces the testing time. Also, only a simple jig, which is a simple extension bar with a magnetic socket, is required in the proposed method. This reduces the setup time within the workspace and also increases the accuracy level of the measurements. Furthermore, mathematical calculations are simple in the proposed method. These facts prove that the proposed method offers a number of improvements that reduce the test time.

7. Conclusion

In this paper, simultaneous five-axis control motion with three ball bar settings was newly proposed. This motion can be used to estimate the inherent deviations in double pivot head type five-axis machining centers. To estimate all ten deviations, three measurements were conducted, two by using the axial motion and the other by radial or tangential motion. For all the motions, only a simple extension bar with a magnetic socket was used as a jig. By using the simultaneous five-axis control motion in the axial direction, seven deviations were estimated; four by means of the observation equation and three by simple geometric relations. The remaining three of the ten deviations were estimated by using radial or tangential direction simultaneous five-axis motion.

The proposed method is superior to the method using the spherical interpolation motion specified in the ISO 10791-6, because the number of measurements is less, the jig for measurement is simple and the mathematical calculation is also simple.

This measurement method will be proposed to the ISO standard relating to the acceptance test for the five-axis machining centers.

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